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Turbulent Natural Convection Modeling Using the FVM and SIMPLEC

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Abstract. The objective of this study is to conduct a numerical investigation of turbulent natural convection in a 3-D cavity using the Finite Volume Method and the SIMPLEC method. The statistical-averaging process of the mass, momentum and energy governing equations introduces unknown turbulent correlations into the mean flow equations which represent the turbulent transport of momentum, heat and mass, namely Reynolds stress $(\overline{u_i u_j})$ and heat flux $(\overline{u_i \theta})$, which are modeled using k-ω SST model. The Reynolds-Averaged Navier-stokes (RANS), energy and k-ω SST turbulent equations are first non-dimensionalized and the resulting equations discretized using Finite Volume Method and solved using SIMPLEC. The results are validated using benchmark experimental results conducted based on Rayleigh number of Ra = 1.58×10^9. From the results, the mean temperature profiles show an almost uniform distribution in the enclosure core, mean vertical velocity profiles are asymmetrical and the mean horizontal velocity profiles show a rise of velocities near the heated surface due to a spike in kinetic energy which outputs an increased convective heat transfer coefficient.

Keywords: Turbulence, natural Convention, k-ω SST model, Finite Volume Method, SIMPLEC Method

Introduction

In fluid dynamics, turbulence is a flow regime characterized by chaotic and stochastic changes. This includes low momentum diffusion, high momentum convection and rapid variation of pressure and velocity in space and time. Turbulent flows exist everywhere in nature from the jet stream to the oceanic currents. Convection flows are one of the fundamental problems in fluid dynamics due to their role in meteorology where they appear as wind as an outcome of solar radiation in the atmosphere and in industrial applications, where they are used in cooling systems to reduce possible noise exposures and technical failures. Many analytical, experimental and numerical investigations have been performed in the past, but the findings are not exhaustive and so these flows are still of substantial interest. The focus of this paper is on computational study of the lift due to the incured convection and the resultant thermal mixing of the flow in an enclosure described below.

Literature Review

Natural turbulent convection in cavities attracts a good amount of interest from thermal scientists given its relevance in industrial and/or civil engineering applications such as energy transfer in rooms and buildings, nuclear reactor cooling, solar collectors and electrical component cooling. A significant number of experimental and theoretical works have been carried out in the past in an attempt to understand turbulent natural convective flows in enclosures.

Among the work is that of Kulacki (1975) who studied natural convection in a horizontal fluid layer boundary bounded by upper isothermal surface and bottom insulated plate for Prandtl numbers varying from 2:75 to 6:85 and Rayleigh numbers up to 2×10^{12}, the experimental data of Kulacki (1975) were correlated by the following expression:

$$Nu_{top} = 0.403Ra^{0.226} \quad (1)$$

Further research on natural convention in an enclosure with localized heating and cooling has been studied by Gatheri *et al.* (1994). Gatheri *et al.* (1994) further investigated how to use False Transient Factors for the Solution of Natural Convection Problems and has well done a parametric Study of an Enclosure with Localized Heating and Cooling, Gatheri (1997). Sigey (2004), not only did research on Numerical Free Convection Turbulent Heat Transfer in an Enclosure but also carried out parametric studies on a rectangular enclosure using the standard k-ε model. Further work on natural turbulent convection has been about the use of mesh generation for the solution of natural convection problems, Gatheri (2005), use of a variable False transient for the solution of Coupled elliptic Equations and use of Buoyancy Driven Natural Convection heat transfer in an enclosure and Magnetohydrodynamic (MHD) free convective flows past an infinite vertical porous plate with Joule heating, Gatheri (2005).

The validation of $\kappa-\varepsilon, \kappa-\omega, SST$ and a coarse DNS models for turbulent natural convection in a differentially heated cavity containing a fluid with P r = 0:71 and Rayleigh numbers ranging from 1.58×10^{9} to 10^{12} was performed by Aounallah *et al*. (2007). The conclusion of Aounallah *et al*. (2007) was that the k-ω-SST provided superior outcomes than the other models analyzed, though it was not able to reproduce accurately the mean flow.

Geniy and Mikhail (2010) researched on turbulent flow convection in a rectangular enclosure with finite thickness thermally- conducting walls with local heating at the base of the enclosure provided that there is convective – radiative heat exchange with surrounding on one of the external borders. The mathematical formulation comprised of the standard two equation K-Ɛ model with wall functions along with the Boussinesq (1903) approximation. Key emphasis was given to the effects of Grashof number, the transient factor and thermal conductivity ratio.

Sheng and Rui (2011) studied the effects of thermal Rayleigh number, ratio of buoyancy forces and aspect ratio on entropy generation of turbulent double – effective natural convection in an enclosure and concluded that the total entropy generation number increases with Rayleigh number.

Seok-Ki Choi et al (2012) did a study on the effects of the treatments of Reynold's stress and accuracy of the solution for free convection in enclosures. The turbulence model considered were k-ε model, SST model and elliptic relaxation (V2 –f) model and elliptic blending second- moment closure (EBM). The three different treatments of the turbulent heat flux are the Generalized Gradient Diffusion Hypothesis (GGDH), the Algebraic Flux Model (AFM) and the differential flux model. Evaluation of turbulence models was performed for Turbulent Natural Convection in a 1:5 rectangular enclosure (Ra = 4.3×10^{10}) and in a square enclosure with conducting top and bottom walls (Ra = 91.58×10^{9}) and the Rey-Leigh – Bernard Convection (Ra = 2×10^{6} ~ Ra = 10^{9}) and their relative performance of the models examined.

Awuor (2012) did a study to assess the performance of three numerical turbulence models; $\kappa-\varepsilon, \kappa-\omega,$ and $\kappa-\omega\ SST$ to find the model with a better approximation to the experimental data in predicting heat transfer profiles due to natural convection inside an air filled cavity. It involves solving the momentum and energy equations using the vorticity-vector potential formulation. This formulation eliminated the need to solve the pressure terms. Numeric data for velocity and temperature distribution was obtained and compared with the simulated data and was noticed that the K-Ɛ Model was not suitable at the boundaries with high temperature gradient but did well in a natural stream flow.

The results by Awuor (2012) further showed that $\kappa - \omega\ SST$ model is a more accurate layer simulation under high temperature gradient as compared with the $\kappa - \varepsilon$ and $\kappa - \omega$ models. A numerical data was then obtained for a test problem using the best model, $\kappa - \omega\ SST$ the resultant numerical data stratified into three regions: a cold upper region, a hot region in the area between the heater and a warm lower region. Whereas Awuor (2012) used Finite Difference Method for discretization and we seek to use Finite Volume Method and moreso SIMPLEC algorithm is this paper.

Mathematical Formulation

In this paper, modeling turbulent heat transfer in a natural convection flow using the Finite Volume Method grid within a cavity is conducted. The geometry consists of a hot surface, located on the left side of the rectangular cavity wall, and a cold surface on the right side. The enclosure is heated on the hot wall (Red color) and cooled on the cold wall (blue color) as illustrated in Figure 1. The measurement of Ampofo and Karyiannis (2003) were used. The walls measures 0.75m by 0.75m wide by 1.5m. The hot and cold walls of the cavity were isothermal at 323±0.15K and 283±0.15K respectively, giving a Reyleigh number of 1.58×10^9. Each of the remaining walls are adiabatic. Fluid flow is to depend only on the temperature difference given as $\Delta T = T_h - T_w$. Aspect ratio is 0.5. Furthermore, the Boussinesq Approximation (1903) is assumed and is presented below. In this research, we will study the variables as used by Ampofo and Karyiannis (2003).

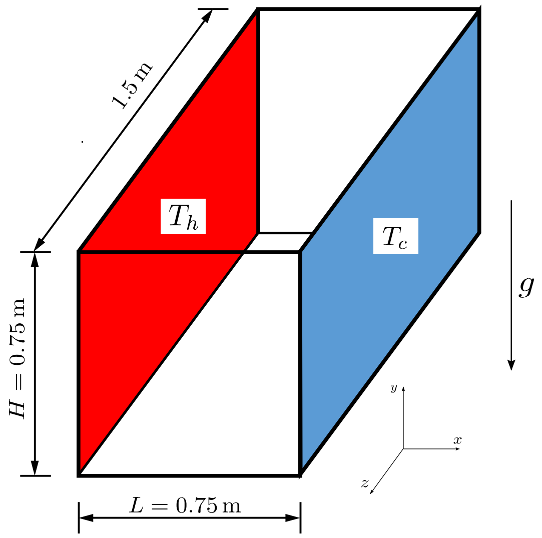

Figure 1. Geometry of the 3-D numerical model

Governing Equations

The resulting equations in general form after non dimensionalisation of the time averaged equations of motion, become:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho U_i + \overline{\rho u_j}\right) \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho U_i + \overline{\rho u_i}\right) + \frac{\partial}{\partial x_j}\left(\rho U_i U_j + U_i \overline{\rho u_j}\right) = -N_1 \frac{\partial P}{\partial x_i} + N_2 p g_i + \frac{\partial}{\partial x_j}\left(N_3 \tau_{ij} - U_i \overline{\rho u_i} - \rho \overline{u_i u_j} - \overline{\rho u_i u_j}\right) = 0 \tag{3}$$

$$\frac{\partial}{\partial t}\left(c_P \rho \Theta + c_P \overline{\rho \Theta}\right) + \frac{\partial}{\partial x_j}\left(c_P \overline{\rho U_j \Theta}\right) = L_1 \left[\frac{\partial p}{\partial t} + U_j \frac{\partial p}{\partial x_j} + \overline{u_j \frac{\partial p}{\partial x_j}}\right] + \frac{\partial}{\partial x_j}\left(L_2 \lambda \frac{\partial \Theta}{\partial x_j} - c_P \overline{\rho \Theta} + c_P \rho \Theta\right) + L_3 \emptyset \tag{4}$$

Space Discretization of the Solution Domain by FVM

The process of space discretization involves dividing the computational domain into a finite number of contagious control volumes, where the resulting statements express the exact conservation of relevant properties for each control volumes.

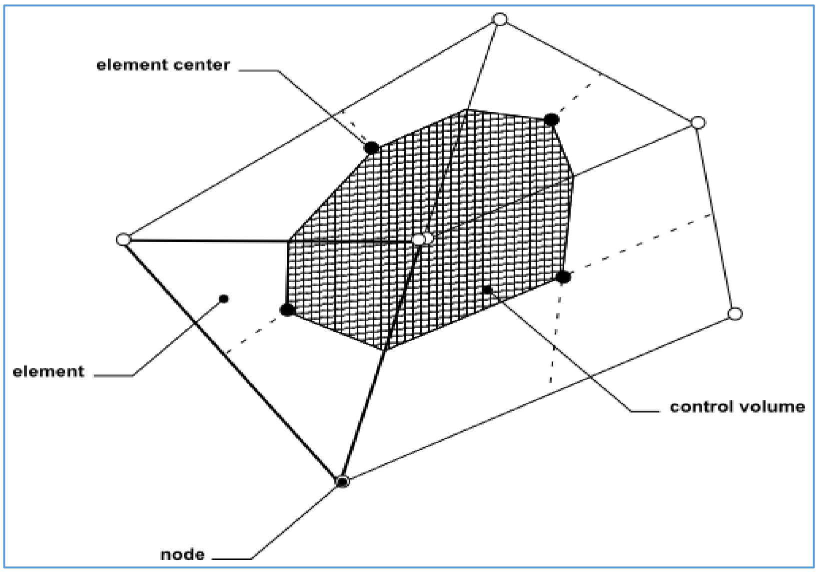

Figure 2. Control-volume element

Figure 2 above shows a typical two-dimensional mesh. Finite Volume Method was preferred to Finite Difference Method for the following reasons:

i) Spatial discretization is totally flexible. Hence you can handle complex geometries, reduce geometric errors and give more resolutions in regions of interest.

ii) FVM naturally conserves variables when applied to PDEs expressing conservation laws since, as two neighboring cells share a common interface. As a result, mass, momentum and energy are conserved even on coarse grids.

iii) This method requires no transformation of equations in terms of body-fitted coordinate system as is required in Finite-Difference Method.

Governing Equations' Discretization Using FVM

Before describing the discretization scheme, choice of arrangement on the grid requires some iluustration. A typical arrangement is depicted in Figure 3, which is in 2-D, for convenience.

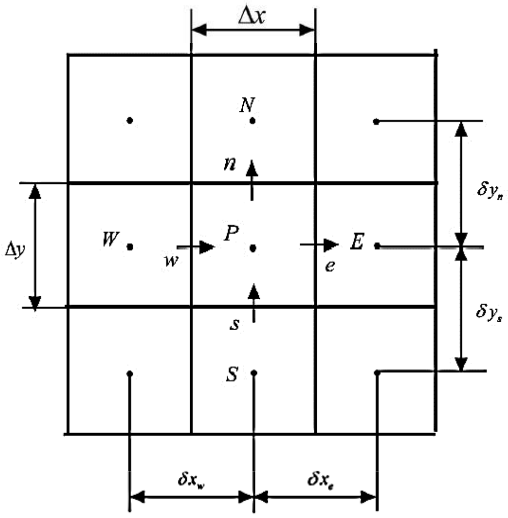

Figure 3. Control volumes in 2D

The discretized continuity equation for the $x-, y-$ and $z-$directions is

$$[(\rho u)_e - (\rho u)_w]\Delta y\Delta z + [(\rho v)_n - (\rho v)_s]\Delta z\Delta x + [(\rho w)_t - (\rho w)_b]\Delta x\Delta y = 0 \quad (5)$$

The discretized momentum equations for the $x-, y-$ and $z-$ directions respectively are

$$a_e u_e = \sum a_{nb} u_{nb} + b + (P_P - P_E)A_e \quad (6)$$

$$a_n v_n = \sum a_{nb} v_{nb} + b + (P_P - P_N)A_n \quad (7)$$

$$a_n w_t = \sum a_{nb} w_{nb} + b + (P_P - P_t)A_t \quad (8)$$

The SIMPLEC Solution Algorithm

The SIMPLEC Algorithm follows the same steps as SIMPLE algorithm, with the difference that the momentum equations are manipulated so that the SIMPLEC velocity correction equations omit terms that are less significant than those omitted in SIMPLE.

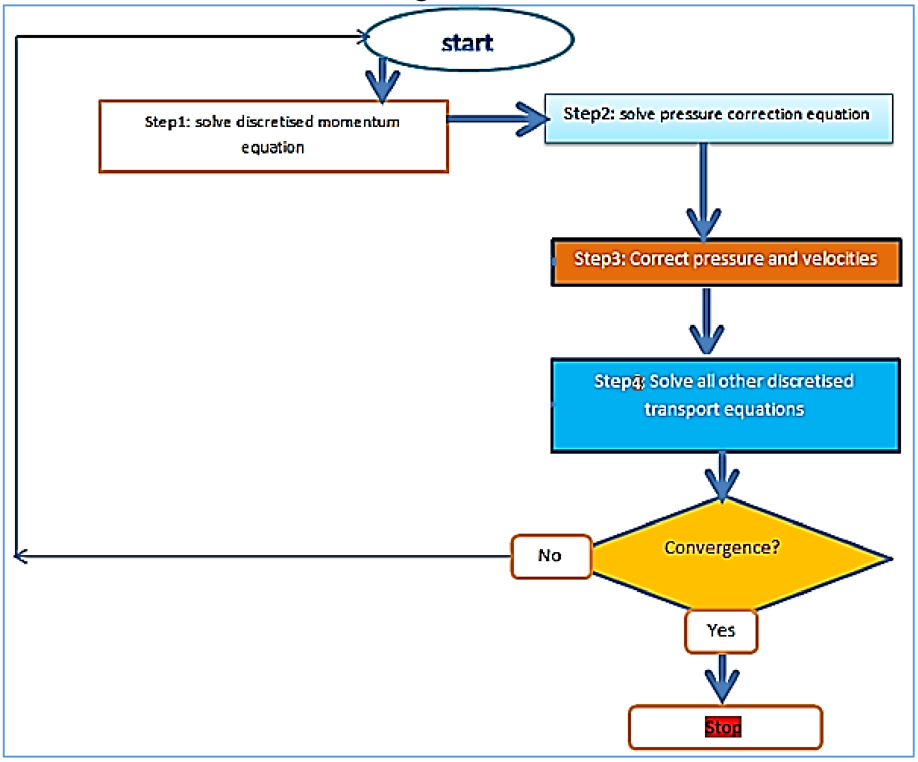

Figure 4. SIMPLEC Algorithm flow chart

Results and Discussion

The results presented here were obtained by solving equations (5), (6), (7) and (8)) by SIMPLEC algorithm after discretization using the Finite Volume Method as shown in Figure 4 and within Boussinesq Approximation (1903) gave the following numerical solutions. The numerical results we have found were validated against the experimental data provided by Ampofo and Karayiannis (2003). This benchmark is at a Rayleigh number of 1.58×10^9.

Solution Convergence by SIMPLEC Method

Convergence was monitored with residuals, whereby a decrease in residuals by three orders of magnitude was to indicate at least qualitative convergence whereby case residual plots would show when the residual values have reached the specified tolerance. For SIMPLEC, the residual convergence criterion for each variable was achieved and the residual imbalance became negligible after 350 iterations in a duration of 1hr, 15min.

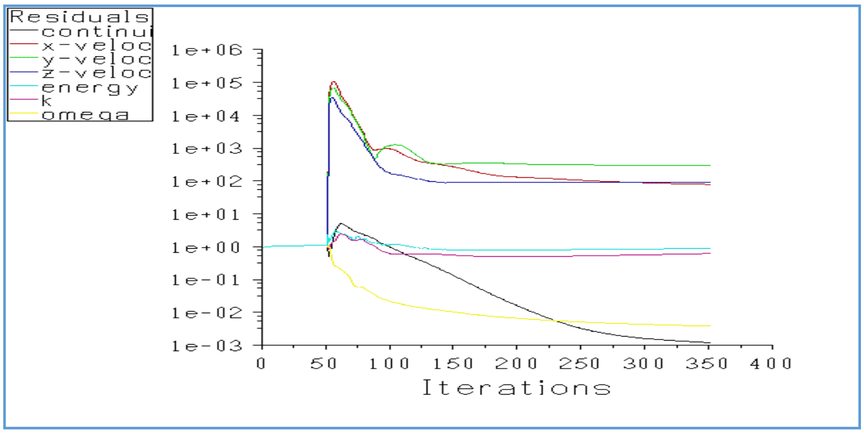

Figure 5. Scaled residuals by SIMPLEC

Validation of Results

Verification and Validation to assess the accuracy and reliability of results in this numerical code was done against the experimental solutions obtained from Ampofo and Karayiannis (2003).

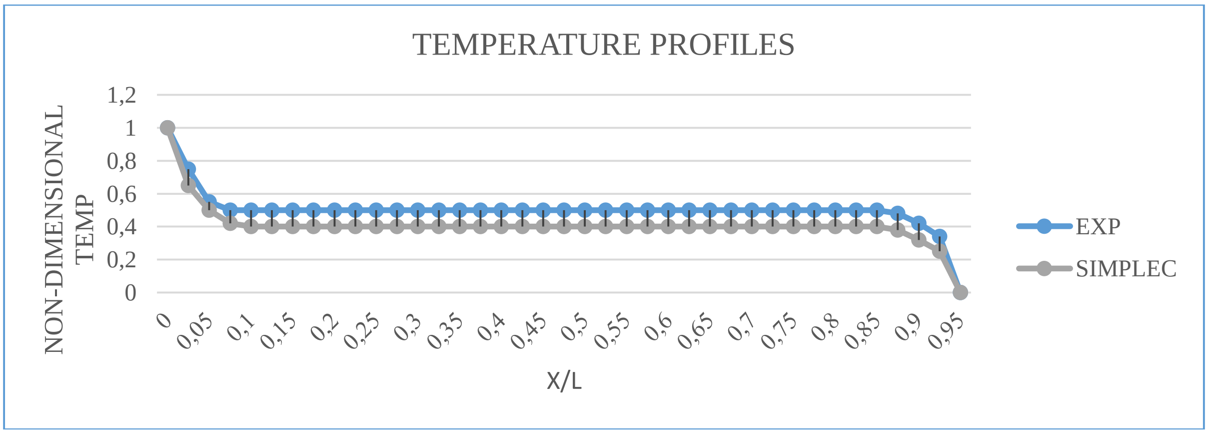

Figure 6. Comparison of the mean temperature at y/H=0.5

Temperature profiles. From Figure 6, the mean temperature profiles show an almost uniform distribution in the enclosure core. This shows that in the enclosure core region, there is very little activity as the mean temperature is nearly uniform. The predicted temperatures by SIMPLEC show a minimum which is lower than the experimental values in the core of the cavity. Again this shows insufficient mixing with the laminar core. In the enclosure core region, there is very little activity; the mean temperature is nearly uniform.

Mean vertical velocity. Figure 7 shows the profiles for rate of change of vertical displacement of the fluid particles with time. The profiles are asymmetrical and with a peak near the heated surface. As seen in Figure 7, there is good agreement between the experimental data and the predicted data in terms of the mean velocity. The peak value of velocity is particularly well captured by SIMPLEC method. In the enclosure core region, there is very little activity and hence the fluid velocity is very small.

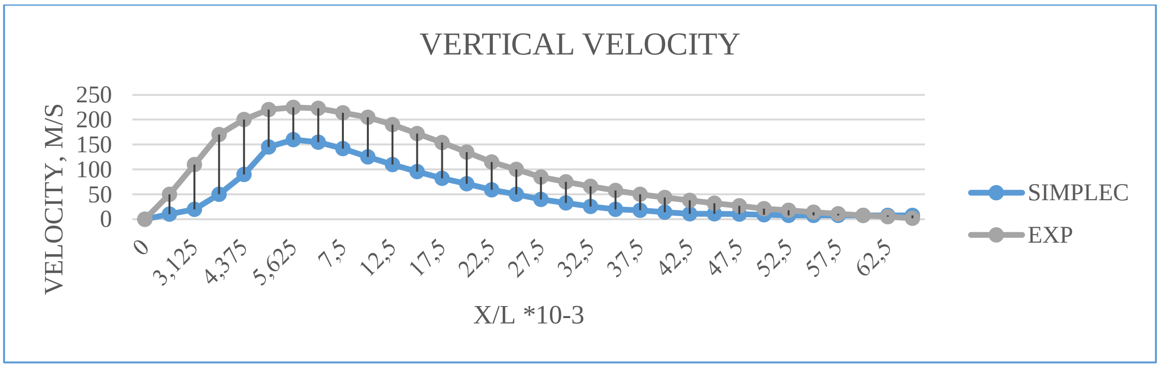

Figure 7. Comparison of the vertical velocity

Mean horizontal velocity. Figure 8 shows the profiles of the rate of change of horizontal displacement of the fluid with time.

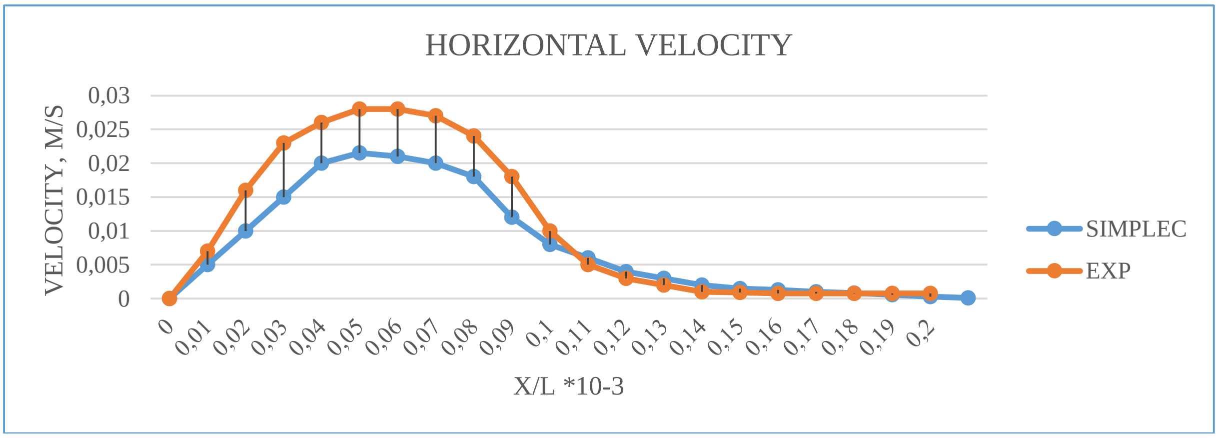

Figure 8. Comparison of the horizontal velocity

The rise of velocities near the heated surface of the cavity is as a result of fluid gaining kinetic energy from the heated wall causes an increased convective heat transfer coefficient, while there is a drop of velocities after 0.04. Generally, there is good agreement between the experimental data and the predicted data in terms of the mean horizontal velocity, as in Figure 8.

Conclusion

i) The mean temperature profiles show an almost uniform distribution in the enclosure core.
ii) Mean vertical velocity profiles are asymmetrical and with a peak near the heated surface.
iii) The mean horizontal velocity profiles show a rise of velocities near the heated surface of the cavity is as a result of fluid gaining kinetic energy from the heated wall causes an

increased convective heat transfer coefficient, while there is a drop of velocities after 0.04.

iv) From the numerical data, the numerical method produced a solution which approached the exact solution by Ampofo and Karayiannis (2003) as the grid spacing reduced to zero.

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