ISSN 2522-9400 **European Modern Studies Journal** Vol 8 No 6
available at www.journal-ems.com
DOI: 10.59573/emsj.8(6).2024.11

Another Approach for Calculus of a Statically Determined Beam with Cantilever and with One Intermediate Support for Application of a Dental Bridge with Mesial Extension and One Missing Tooth through the Transfer-Matrix Method

Mihaela SUCIU
Department of Mechanical Engineering,
Technical University of Cluj-Napoca, Romania

Abstract. The problem of continuous beams with intermediate support treated through the Transfer-Matrix Method is very special and interesting. This paper presents a study for an application of a statically determinate beam with cantilever and with an intermediate support. This approach is applied in orthodontics. The application of this research was made for a case study of a dental bridge with two adjacent poles and with only one missing tooth, because dental restorations are very important not only from an aesthetic point of view, but especially for mastication and therefore, for the health of the entire human body. A case study was presented of a dental bridge in mesial extension assimilated with a statically determined beam with cantilever and with an intermediate support. The two poles of the dental bridge are assimilated as the edges of the beam and the missing tooth is the cantilever. The calculus and results were validated with those from the classical calculus in the Strength of Materials. In the future, through the Transfer-Matrix Method, other new and original case studies with different modeling and requests will be presented.

Key words: dental bridge, mesial extension, statically determinate beam with cantilever, intermediate support, state vector, Transfer-Matrix Method

Introduction

The Transfer-Matrix Method (TMM) is a different approach in many fields where iterative calculus and optimized results according to different criteria are required. In this sense, interdisciplinary studies are necessary. A new, original and elegant theoretical calculus for a statically determinate beam with cantilever and with an intermediate support charged with an uniform load along its entire length is presented in this work. The application of this research was made for case study of a dental bridge with two adjacent poles and with only one missing tooth, because dental restorations are very important not only from an aesthetic point of view, but especially for mastication and therefore, for the health of the entire human body. The calculus and results were validated with those from the classical calculus in the Strength of Materials. In the future, through the TMM, other new and original case studies with different modeling and requests will be presented.

Literature Review

In scientific literature, different approaches are presented, both mathematical, theoretical and experimental. A comprehensive review on dental calculus can be found (Aghanashini et al., 2016) and a systematic review and meta-analysis about biological oriented preparation technique (BOPT) for tooth preparation is presented (Al-Haddad et al., 2024). It can be shown a radiographic evaluation of the margins of clinically acceptable metal-ceramic crowns (Badar et al., 2022). There is a study on ten-year survival of bridges placed in the General Dental Services in England and Wales (Burke & Lucarotti, 2012). A recent development in beta titanium alloys for biomedical applications is presented (Chen, Cui & Zhang, 2020). It can be studied about the Transfer-Matrix Method for calculus of long cylinder tube with industrial applications (Codrea et al., 2023) and about calculus through the Transfer Matrix Method of a beam with intermediate support with applications in dental restorations (Cojocariu-Oltean et al., 2024). It is presented an experimental investigation about bending fracture of Co-Cr dental bridges, produced by additive technologies (Dikova, 2018) and a simulation analysis and test for bending fracture of Co-Cr dental bridges, produced by additive technologies (Dikova & Vasilev, 2019). Dimensional accuracy and surface roughness of polymeric dental bridges produced by different 3D printing processes is given (Dikova et al, 2018). About integrated construction and simulation of tool paths for milling dental crowns and bridges is presented (Gaspar & Weichert, 2023). It is very important to know the basis of calculus through the Transfer-Matrix Method for the calculus of structures (Gery & Calgaro, 1987). An interesting review about the dental bridge procedure to straighten loose teeth is given (Ifwandi, 2023). A potential application of materials based on a polymer and CAD/CAM composite resins in prosthetic dentistry is presented (Jovanovic, Živic & Milosavljevic, 2021). About the buckling of piles in layered soils by Transfer-Matrix Method is shown (Lu, Tao & Quian, 2021). An in vitro study about the fracture resistance of CAD/CAM provisional crowns with two different designs is presented (Mekled et al., 2024). It can be found a comparison of tensile bond strength of fixed-fixed versus cantilever single-and double-abutted resin-bonded bridges dental prosthesis (Narwani et al., 2022). Effect of endodontic access preparation on fracture load of translucent versus conventional zirconia crowns with varying occlusal thickness is given (Nejat et al., 2021). Titanium-based biomaterials for preventing stress shielding between implant devices and bone is presented (Niinom & Nakai, 2011). Studies about mechanical reliability, fatigue strength and survival analysis of new polycrystalline translucent zirconia ceramics for monolithic restorations are shown (Pereira et al., 2018). The design implications from fatigue of zirconia and dental bridge geometry are given (Quinna et al., 2010). It is presented current and emerging applications of 3D printing of dental prostheses (Rezaie et al., 2023). About the effect of preparation taper on the resistance to fracture of monolithic zirconia crowns can be found (Schriwer et al., 2021) and the effect of margin design on fracture load of zirconia crowns too (Skjold, Schriwer & Øilo, 2019). A study about the fatigue of zirconia under cyclic loading in water and its implications for the design of dental bridges is given (Studart et al., 2007). An approach by TMM for mandible body bone calculus is given (Suciu, 2023). Basis of classical calculus of strength of materials it can be found (Suciu & Tripa, 2021). The calculus through the TMM for elastic-buckling analysis of compression bar is presented (Sun & Li, 2011). A review of the surface modifications of titanium alloys for biomedical applications is presented (Uporabo, 2017). Research on the surface of the dental alloys with cobalt-crom base is shown (Uriciuc, Vermesan & Botean, 2019). Formulas for stress and strain are given (Warren, 1989).

Methods

General approach with TMM for beams with intermediate supports

A statically beam with intermediate supports can be studied as an indeterminate continuous beam, (Gery & Calgaro, 1987). For starters, it is considered a statically indeterminate continuous beam with cantilever at the right extremity, articulate at the left edge and free at the right end at the end of the console and with intermediate supports, as in Figure 1.

Calculus hypotheses

It is considered a statically indeterminate continuous beam, articulate at the left end, free at the right end and with i $(i=1, n-1)$ intermediate simple supports, as in Figure 1.

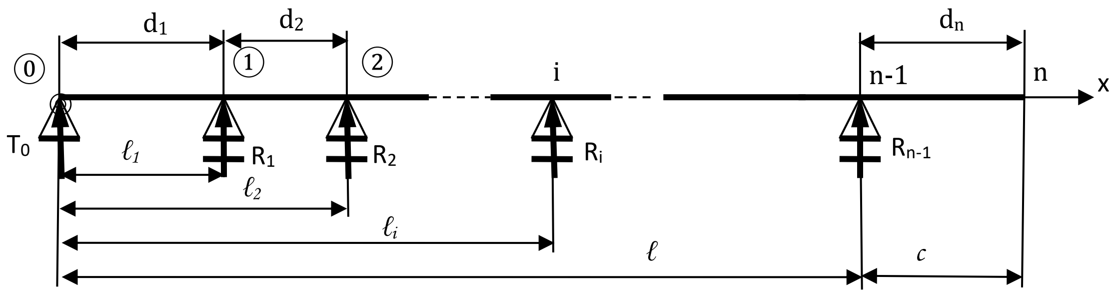

Figure 1. Indeterminate beam with cantilever and with *i (i=1, n-1)* intermediate supports

For all the beam, the lengths of all parts are known.

It is considered the constant inertia of the entire beam.

A reference system with the origin in the left end, the end noted with *0*, it is considered.

Generally, the total external vertical forces are characterized by a total charge density *q(x)*, as (1):

$$q(x) = q_1(x) + q_2(x) + q_3(x) \tag{1}$$

That consists of:

- $q_1(x)$ corresponds to the vertical concentrated loads;
- $q_2(x)$ corresponds to the force uniformly distributed over the entire beam;
- $q_3(x)$ corresponds to the unknown vertical reactions arise at each edge, at the intermediate supports, i.e. vertical reactions due to the external forces considered to act on the beam.

In the approach that will be presented, the unknown vertical reactions arise from each intermediate support and at the left end too, will be considered as external forces.

State vectors for some section of continuous beam

It is defined the state vectors for different sections of the continuous beam can as follows.

*{**S**(x)}* is the ***S***tate vector at the section *x* on the *x*-axis, the position of the section being measured from the origin of the reference system, which is in the left edge *0*, as (2):

$$\{S(x)\} = \{f(x), r(x), M(x), T(x)\}^{-1} \tag{2}$$

The elements of the state vector (2) are:

- *f(x)* - the arrow in the section *x*;
- *r(x)* - the rotating in the section *x*;
- *M(x)* - the bending moment in the section *x*;
- *T(x)* - the cutting force in section *x*.

The state vector at the origin $\{S(0)\}= \{S\}_0$ in the section *0* is as (3):

$$\{S(0)\} = \{f(0), r(0), M(0), T(0)\}^{-1} = \{S\}_0 = \{f_0, r_0, M_0, T_0\}^{-1} \tag{3}$$

l, the total length of continuous beam with cantilever, is as (4):

$$l = \sum_{i=1}^{n-1} d_i \tag{4}$$

when $d_i, i=1, n-1$ is the distance between two consecutive supports and $d_n = c$ is the length of the console at the right end of the beam. At the last section, in the section *n*, the state vector $\{S(l)\} = \{S(l)\}= \{S\}_l$ for $x=l+c$, is as (5):

$$\{S(l+c)\} = \{f(l+c), r(l+c), M(l+c), T(l+c)\}^{-1} = \{S\}_{l+c} =$$
$$= \{f_{l+c}, r_{l+c}, M_{l+c}, T_{l+c}\}^{-1} \tag{5}$$

Transfer-Matrix at some section *x*

It is noted with $[TM]_x$ the Transfer-Matrix for some section *x* is as (6), (Gery & Calgaro, 1987):

$$[TM]_x = \begin{bmatrix} 1 & x & \frac{x^2}{2EI} & -\frac{x^3}{6EI} \\ 0 & 1 & \frac{x}{EI} & -\frac{x^2}{2EI} \\ 0 & 0 & 1 & -x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Matrix relation for some section *x* of continuous beam with cantilever

The state vector from the origin (the section *0*) is connected with the state vector from some section *x*, with relation (7):

$$\begin{Bmatrix} f(x) \\ r(x) \\ M(x) \\ T(x) \end{Bmatrix} = [TM]_x \begin{Bmatrix} f(0) \\ r(0) \\ M(0) \\ T(0) \end{Bmatrix} + \{E(x)\} \tag{7}$$

when:

- $[TM]_x$ is the ***T**ransfer-**M**atrix* corresponding at section *x*;
- $\{E(x)\}$ is the vector for ***E**xterior* forces at section *x*.

The elements of vector $\{E(x)\}$ will be the proper external forces and the unknown vertical reactions from the supports.

Vector for exterior efforts at some section *x*

The vector for exterior efforts $\{E(x)\}$ is composed of the effective external forces and the still unknown reactions.

Charge Density for a continuous beam

Next, the three terms from relation (1) will be explained. With Dirac's and Heaviside's functions and operators, (Gery & Calgaro, 1987), the charge densities can be written:

- for the exterior vertical concentrated loads, the charge density $q_1(x)$, for a number *j* of exterior loads V_j, $j=1, k$, acting in the point c_j, $j=1, k$, can be written as (8):

$$q_1(x) = -\sum_{j=1}^{k} V_j\, \delta\left(x - c_j\right) \tag{8}$$

- for the uniformly distributed vertical load $u(x)$, if it is on the whole beam, the charge density $q_2(x)$, can be written as (9):

$$q_2(x) = -u(x)\delta(x) \tag{9}$$

- for the vertical reactions, R_i, $i=0, n-1$ in the edges, the density charge $q_3(x)$, can be written as (10):

$$q_3(x) = \sum_{i=0}^{n-1} R_i\, \delta(x - l_i) \tag{10}$$

with the following specifications (11):

$$\begin{cases} l_0 = 0 \\ l_{n-1} = l \\ R_0 = T_0 \end{cases} \tag{11}$$

and:

- *j* is the point where act the concentrated vertical force V_j;
- V_j, $j=1, k$, are the exterior vertical concentrated loads;
- *k* is the number of the concentrated vertical force;
- *i* refers to the edge with number *i;*
- *n* is the total number of edges;
- $u(x)$ is the uniformly vertical distributed load for all beam;
- R_i is the **v**ertical reaction in the edge *i*.

The mathematical development calculus with Dirac's and Heaviside's functions and operators for (8), gives successively (12):

$$\begin{cases} q_{1_1}(x) = -\sum_{j=1}^{k} V_j\, Y\left(x - c_j\right) \\ q_{1_2}(x) = -\sum_{j=1}^{k} V_j\left(x - c_j\right) Y\left(x - c_j\right) \\ q_{1_3}(x) = -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^2}{2} Y\left(x - c_j\right) \\ q_{1_4}(x) = -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^3}{6} Y\left(x - c_j\right) \end{cases} \tag{12}$$

For (9) it is obtained (13):

$$\begin{cases} q_{2_1}(x) = -xu(x)Y(x) \\ q_{2_2}(x) = -\frac{x^2}{2} u(x)Y(x) \\ q_{2_3}(x) = -\frac{x^3}{6} u(x)Y(x) \\ q_{2_4}(x) = -\frac{x^4}{24} u(x)Y(x) \end{cases} \tag{13}$$

And for (10) it is obtained (14):

$$\begin{cases} q_{3_1}(x) = \sum_{i=0}^{n-1} R_i\, Y(x - l_i) \\ q_{3_2}(x) = \sum_{i=0}^{n-1} R_i\,(x - l_i)Y(x - l_i) \\ q_{3_3}(x) = \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^2}{2} Y(x - l_i) \\ q_{3_4}(x) = \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^3}{6} Y(x - l_i) \end{cases} \tag{14}$$

The total charge density can be written as (15):

$$\begin{cases} q'(x) = -\sum_{j=1}^{k} V_j\, Y\left(x - c_j\right) - xu(x)Y(x) + \sum_{i=0}^{n-1} R_i\, Y(x - l_i) \\ q''(x) = -\sum_{j=1}^{k} V_j\left(x - c_j\right)Y\left(x - c_j\right) - \frac{x^2}{2} u(x)Y(x) + \sum_{i=0}^{n-1} R_i\,(x - l_i)Y(x - l_i) \\ q'''(x) = -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^2}{2} Y\left(x - c_j\right) - \frac{x^3}{6} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^2}{2} Y(x - l_i) \\ q''''(x) = -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^3}{6} Y\left(x - c_j\right) - \frac{x^4}{24} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^3}{6} Y(x - l_i) \end{cases} \tag{15}$$

Vector of exterior forces at some section *x*

For exterior forces at some section x, the vector of exterior loads (including the vertical reactions) is as (16):

$$\{E(x)\} = \\ = \begin{Bmatrix} -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^3}{6} Y\left(x - c_j\right) - \frac{x^4}{24} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^3}{6} Y(x - l_i) \\ -\sum_{j=1}^{k} V_j \frac{\left(x - c_j\right)^2}{2} Y\left(x - c_j\right) - \frac{x^3}{6} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x - l_i)^2}{2} Y(x - l_i) \\ -\sum_{j=1}^{k} V_j\left(x - c_j\right)Y\left(x - c_j\right) - \frac{x^2}{2} u(x)Y(x) + \sum_{i=0}^{n-1} R_i\,(x - l_i)Y(x - l_i) \\ -\sum_{j=1}^{k} V_j\, Y\left(x - c_j\right) - xu(x)Y(x) + \sum_{i=0}^{n-1} R_i\, Y(x - l_i) \end{Bmatrix} \tag{16}$$

Calculus algorithm through the TMM

The matrix relation (7) can be written as (17):

$$\begin{Bmatrix} f(x) \\ r(x) \\ M(x) \\ T(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{x^2}{2EI} & -\frac{x^3}{6EI} \\ 0 & 1 & \frac{x}{EI} & -\frac{x^2}{2EI} \\ 0 & 0 & 1 & -x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} f(0) \\ r(0) \\ M(0) \\ T(0) \end{Bmatrix} +$$

$$+\begin{Bmatrix} -\sum_{j=1}^{k} V_j \frac{(x-c_j)^3}{6} Y(x-c_j) - \frac{x^4}{24} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x-l_i)^3}{6} Y(x-l_i) \\ -\sum_{j=1}^{k} V_j \frac{(x-c_j)^2}{2} Y(x-c_j) - \frac{x^3}{6} u(x)Y(x) + \sum_{i=0}^{n-1} R_i \frac{(x-l_i)^2}{2} Y(x-l_i) \\ -\sum_{j=1}^{k} V_j (x-c_j) Y(x-c_j) - \frac{x^2}{2} u(x)Y(x) + \sum_{i=0}^{n-1} R_i (x-l_i) Y(x-l_i) \\ -\sum_{j=1}^{k} V_j Y(x-c_j) - x u(x)Y(x) + \sum_{i=0}^{n-1} R_i Y(x-l_i) \end{Bmatrix} \quad (17)$$

For $x=l+c$, for the last section, the expression (17) can be written as (18):

$$\begin{Bmatrix} f(l+c) \\ r(l+c) \\ M(l+c) \\ T(l+c) \end{Bmatrix} = \begin{bmatrix} 1 & l+c & \frac{(l+c)^2}{2EI} & -\frac{(l+c)^3}{6EI} \\ 0 & 1 & \frac{l+c}{EI} & -\frac{(l+c)^2}{2EI} \\ 0 & 0 & 1 & -(l+c) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} f(0) \\ r(0) \\ M(0) \\ T(0) \end{Bmatrix} +$$

$$+\begin{Bmatrix} -\sum_{j=1}^{k} V_j \frac{(l+c-c_j)^3}{6} Y(l+c-c_j) - \frac{(l+c)^4}{24} u(l+c) + \sum_{i=0}^{n-1} R_i \frac{(l+c-l_i)^3}{6} Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j \frac{(l+c-c_j)^2}{2} Y(l+c-c_j) - \frac{(l+c)^3}{6} u(l+c) + \sum_{i=0}^{n-1} R_i \frac{(l+c-l_i)^2}{2} Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j (l+c-c_j) Y(l+c-c_j) - \frac{(l+c)^2}{2} u(l+c) + \sum_{i=0}^{n-1} R_i (l+c-l_i) Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j Y(l+c-c_j) - (l+c) u(l+c) + \sum_{i=0}^{n-1} R_i Y(l+c-l_i) \end{Bmatrix} \quad (18)$$

In matrix relation (18), conditions for the extreme edges can be placed, in section *0*, as (19):

$$\begin{cases} f_0 = 0 \\ M_0 = 0 \end{cases} \quad (19)$$

and in section n for $x=l+c$, conditions are as (20):

$$\begin{cases} M(l+c) = M_n = 0 \\ T(l+c) = T_n = 0 \end{cases} \quad (20)$$

if there are no concentrated external forces or concentrated bending moments on the end of the console (in the section n).

With the conditions (19) and (20), the matrix relation (18) can be written as (21):

$$\begin{Bmatrix} f(l+c) \\ r(l+c) \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & l+c & \frac{(l+c)^2}{2EI} & -\frac{(l+c)^3}{6EI} \\ 0 & 1 & \frac{l+c}{EI} & -\frac{(l+c)^2}{2EI} \\ 0 & 0 & 1 & -(l+c) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ \omega_0 \\ 0 \\ T_0 \end{Bmatrix} +$$

$$+ \begin{cases} -\sum_{j=1}^{k} V_j \frac{(l+c-c_j)^3}{6} Y(l+c-c_j) - \frac{(l+c)^4}{24} u(l+c) + \sum_{i=0}^{n-1} R_i \frac{(l+c-l_i)^3}{6} Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j \frac{(l+c-c_j)^2}{2} Y(l+c-c_j) - \frac{(l+c)^3}{6} u(l+c) + \sum_{i=0}^{n-1} R_i \frac{(l+c-l_i)^2}{2} Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j (l+c-c_j) Y(l+c-c_j) - \frac{(l+c)^2}{2} u(l+c) + \sum_{i=0}^{n-1} R_i (l+c-l_i) Y(l+c-l_i) \\ -\sum_{j=1}^{k} V_j Y(l+c-c_j) - (l+c) u(l+c) + \sum_{i=0}^{n-1} R_i Y(l+c-l_i) \end{cases} \quad (21)$$

The conditions for the arrows in each intermediate supports must be set, they must be *null* in each intermediate supports as (22):

$$\text{for } i=1, n-1 \Rightarrow f_i = f(l_i) = 0 \quad (22)$$

what gives (23):

$$l_i r_0 - l_i^3 \frac{T_0}{6EI} + \sum_{i=1}^{n-1} (l_i - l_1)^3 \frac{R_i}{6EI} = -\sum_{i=1}^{n} \sum_{j=1}^{k} V_j \frac{(l_i - l_1)^3}{6EI} Y(l_i - l_1) - \sum_{i=0}^{n} \frac{(l_i - l_1)^4}{24EI} u(l_i - l_1) \quad (23)$$

The conditions for *the right* end of the beam, for $x=l+c$, are (20) and give (24):

$$\begin{cases} l_i r_0 - (l+c)T_0 + \sum_{i=0}^{n-1} R_i (l+c-c_j) - \sum_{i=1}^{n} \sum_{j=1}^{k} V_j (l+c-c_j) - \sum_{i=0}^{n} \frac{(l+c-l_i)^2}{2EI} u(l+c-l_i) = 0 \\ -\frac{(l+c)^2}{2EI} T_0 + \sum_{i=1}^{n-1} R_i - \sum_{j=1}^{k} V_j - \sum_{i=0}^{n} (l+c-l_i) u(l+c-l_i) = 0 \end{cases} \quad (24)$$

or (25):

$$\begin{cases} l_i r_0 - (l+c)T_0 + \sum_{i=0}^{n-1} R_i (l+c-c_j) = \sum_{i=1}^{n} \sum_{j=1}^{k} V_j (l+c-c_j) + \sum_{i=0}^{n} \frac{(l+c-l_i)^2}{2EI} u(l+c-l_i) \\ -\frac{(l+c)^2}{2EI} T_0 + \sum_{i=1}^{n-1} R_i = \sum_{j=1}^{k} V_j + \sum_{i=0}^{n} (l+c-l_i) u(l+c-l_i) \end{cases} \quad (25)$$

The relations (23) and (24) can be written in a matrix relation like (25), considering that r_0, $\frac{T_0}{6EI}$ and $\sum_{i=1}^{n-1} \frac{R_i}{EI}$ are the unknowns:

$$\begin{bmatrix} l_1 & -l_1^3 & 0 & 0 & 0 & \cdots & 0 \\ l_2 & -l_2^3 & (l_2-l_1)^3 & 0 & 0 & \cdots & 0 \\ l_3 & -l_3^3 & (l_3-l_1)^3 & (l_3-l_2)^3 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l_i & -l_i^3 & (l_i-l_1)^3 & (l_i-l_2)^3 & (l_i-l_3)^3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l+c & -(l+c)^3 & (l+c-l_1)^3 & (l+c-l_2)^3 & (l+c-l_3)^3 & \cdots & (l+c-l_{n-1})^3 \\ 0 & -(l+c) & l+c-l_1 & l+c-l_2 & l+c-l_3 & \cdots & l+c-l_{n-1} \end{bmatrix} \begin{Bmatrix} r_0 \\ \frac{T_0}{6EI} \\ \frac{R_1}{6EI} \\ \vdots \\ \frac{R_{n-2}}{6EI} \\ \frac{R_{n-1}}{6EI} \end{Bmatrix}$$

$$= \begin{Bmatrix} -\sum_{j=1}^{k} V_j \frac{(l_1)^3}{6EI} Y(l_1) - \frac{(l_1)^4}{24EI} u(l_1) \\ -\sum_{j=1}^{k} V_j \frac{((l_2-l_1))^3}{6EI} - \frac{((l_2-l_1))^4}{24EI} u(l_2-l_1) \\ \vdots \\ -\sum_{i=1}^{n-1}\sum_{j=1}^{k} V_j \frac{(l_i-l_1)^3}{6EI} - \sum_{i=0}^{n} \frac{(l_i-l_1)^4}{24EI} u(l_i-l_1) \\ \vdots \\ \sum_{i=1}^{n-1}\sum_{j=1}^{k} V_j (l+c-l_i) - \sum_{i=0}^{n} \frac{(l+c-l_i)^2}{2EI} u(l+c-l_i) \\ \sum_{j=1}^{k} V_j - \sum_{i=0}^{n} (l+c-l_i) u(l+c-l_i) \end{Bmatrix} \quad (26)$$

The system of linear equations (24) can be solved and the unknowns r_0, T_0 and R_i, $i=1, n-1$ can be determined. Now, the elements of the state vectors for all sections can be calculated, knowing the elements of the state vector from the origin. Then, it is possible to calculus all efforts and deformations in all sections of the beam.

Results and Discussion

Application for a dental bridge in mesial extension with two poles and one edentulous, assimilated as a statically determined continuous beam, with an intermediate support and with cantilever

The mathematical algorithm presented can be applied in dentistry, in occurrence, it will be presented for a dental bridge in mesial extension with two poles and one missing tooth. The dental bridge will be assimilated as a continuous beam, statically determined with an intermediate support and with cantilever.

The dental bridge in mesial extension

It is considered a dental bridge in mesial extension with two poles and one missing tooth, case presented as in Figure 2.

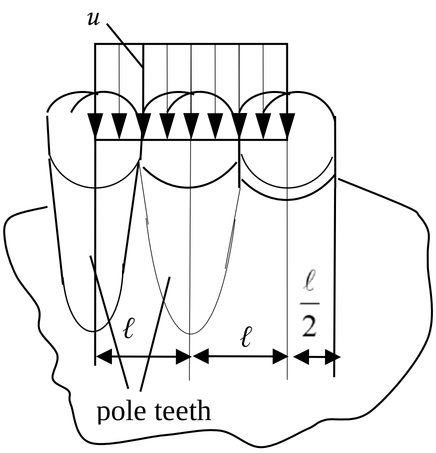

Figure 2. A dental bridge in mesial extension with two poles and one missing tooth

It is proposed a dental bridge with the following characteristics:

- the dental bridge consists of two poles, which can be two consecutive natural teeth prepared to dental bridge as poles, or two consecutive dental implants;
- the second pole is considered as an intermediate support, because it is found between the two extremities of the dental bridge;
- the missing tooth is considered to be at the right end of the dental bridge;
- the distances between the middle of the left pole and the middle of the next pole (second pole) and the distance between the middle of the second pole and the middle of the missing tooth are considered equal each other;
- an uniformly distributed vertical force is considered to act on the dental bridge, as in Figure 2. and Figure 3., as a result of the action of the antagonistic teeth on the jaw.

The continuous beam with cantilever and with one intermediate support

The dental bridge in mesial extension with two poles and one missing tooth, case presented as in Figure 2., can be assimilated as a continuous beam with cantilever and one intermediate support, as in Figure 3.

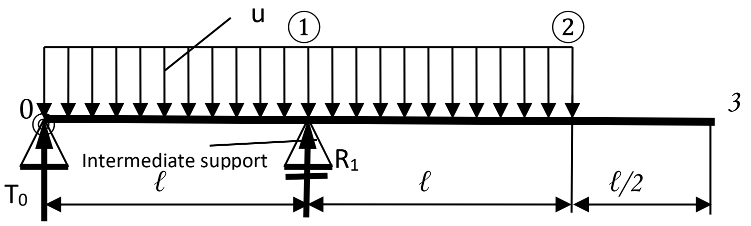

Figure 3. A dental bridge in mesial extension with two poles and one edentulous, assimilated as a determinate beam with cantilever and with an intermediate support

The following conditions are considered to be able to use the calculus algorithm presented***:***

- the beam from Figure 3 is a statically determinate beam;
- it is noted with *l* the distance between the middle of the left pole which is considered as the left edge of the beam and the middle of the second pole which is considered as the intermediate support of the beam, and the distance between the middle of the second pole (the intermediate support) and the middle of the tooth which replaces the missing tooth is *l* too;
- the distance between the middle of the tooth which replaces the missing tooth, and the free extremity is *l*/2;
- for the entire beam, the constant inertia is also considered;
- the cantilever of the beam has the length *3l/2*, that is considered the distance between the middle of the second pole (the intermediate support) and the free end of cantilever;
- the origin of a reference system it is considered in the left edge, the edge *0*;
- the left edge it is considered an articulated edge (in the left pole) and with a simple support in the middle (the intermediate support) and free at the right extremity, the cantilever extremity;
- the uniform distributed vertical load act on the dental bridge and on the beam too, as in Figure 2 and Figure 3.

Approach of calculus for statically determinate continuous beam with cantilever and with an intermediate support through the TMM

The dental bridge from Figure 2. is assimilated as a continuous beam from Figure 3., that is considered for this study.

It is a continuous beam, articulate at the left edge, with a simple support at the right edge and with a simple intermediate support. The simple support is considered as an intermediate support because it is between the two ends of the continuous beam, between the articulate edge of the left side and the simple support at the right side, as in Figure 3.

The left edge is noted as *0*, i.e. the articulate edge will be called the origin section, considering *0* as the origin of the reference system *Ox*. The second edge, the simple intermediate support, is noted with *1* and the right edge is noted with *2*, the section up to which the vertical uniformly distributed load acts.

The charge density $q_2(x)$ corresponding of exterior uniform distributed vertical load for the continuous beam from Figure 3., is as (27):

$$q_2(x) = -u(x)\delta(x) \tag{27}$$

and applying the mathematical calculus with Dirac's and Heaviside's functions and operators and the matrix calculus presented, for the case of Figure 3., the relation (26) can be written as the matrix relation (28):

$$\begin{bmatrix} l & -l^3 & 0 \\ 2l & -(2l)^3 & l^3 \\ 0 & -2l & l \end{bmatrix} \begin{Bmatrix} r_0 \\ \frac{T_0}{6EI} \\ \frac{R_1}{6EI} \end{Bmatrix} = \begin{Bmatrix} \frac{ul^4}{24EI} \\ \frac{u(2l)^4}{24EI} \\ \frac{u(2l)^2}{12EI} \end{Bmatrix} \tag{28}$$

or, (29):

$$\begin{bmatrix} l & -l^3 & 0 \\ 2l & -8l^3 & l^3 \\ 0 & -2l & l \end{bmatrix} \begin{Bmatrix} r_0 \\ \frac{T_0}{6EI} \\ \frac{R_1}{6EI} \end{Bmatrix} = \begin{Bmatrix} \frac{ul^4}{24EI} \\ \frac{2ul^4}{3EI} \\ \frac{ul^2}{3EI} \end{Bmatrix} \tag{29}$$

Due to the fact that the beam with the cantilever from Figure 3. has the cantilever equal to the distance between the supports, $c = l$ and due to the symmetry of the load, additional conditions must be put, for the left support, as (3):

$$T_0 = 0 \tag{30}$$

With the condition from (30), the matrix relation (29) can be written as (31):

$$\begin{bmatrix} l & -l^3 & 0 \\ 2l & -8l^3 & l^3 \\ 0 & -2l & l \end{bmatrix} \begin{Bmatrix} r_0 \\ 0 \\ \frac{R_1}{6EI} \end{Bmatrix} = \begin{Bmatrix} \frac{ul^4}{24EI} \\ \frac{2ul^4}{3EI} \\ \frac{ul^2}{3EI} \end{Bmatrix} \tag{31}$$

or, it can be written under the form (32):

$$\begin{bmatrix} l & 0 \\ 0 & l \end{bmatrix} \begin{Bmatrix} r_0 \\ \frac{R_1}{6EI} \end{Bmatrix} = \begin{Bmatrix} \frac{ul^4}{24EI} \\ \frac{ul^2}{3EI} \end{Bmatrix} \tag{32}$$

(32) can be developed as (33):

$$\begin{cases} lr_0 = \frac{ul^4}{24EI} \\ \frac{R_1}{6EI} = \frac{ul^2}{3EI} \end{cases} \tag{33}$$

with solution (34):

$$\begin{cases} r_0 = \frac{ul^3}{24EI} \\ R_1 = 2ul \end{cases} \tag{34}$$

value identical to that given by the classic calculation from Materials Resistance, (Suciu & Tripa, 2021; Warren, 1989).

Conclusion

The problem of intermediate supports for a statically determinate or indeterminate beams through the TMM is a very interesting approach with Dirac's and Heaviside's functions and operators. This study presents a new and original calculus for the statically determinate continuous beams with intermediate support through the TMM. To derive the general calculus formulas, it is started from the generalization of a statically indeterminate continuous beam, with *n-1* intermediate supports presented. Then it was customized for a dental bridge in mesial extension, as in Figure 2., assimilated with a statically determinate beam with cantilever and with an intermediate support, as in Figure 3. In the presented case study, the results were validated with those obtain from the classical calculus of Strength of Materials. In the future, other new and original cases of applications in orthodontics will be presented.

References

Aghanashini, S., Puvvalla, B., Mundinamane, D. B., Apoorva, S. M., Bhat, D. & Lalwani, M. A. (2016). Comprehensive Review on Dental Calculus. *Journal of Health Sciences & Research*, *7*(2), 42-50. https://doi.org/10.5005/jp-journals-10042-1034.

Al-Haddad, A., Arsheed, N. A. A., Yee, A. & Kohli, S. (2024). Biological oriented preparation technique (BOPT) for tooth preparation: A systematic review and meta-analysis. *Saudi Dental Journal*, *36*(1), 11-19. https://doi.org/10.1016/j.sdentj.2023.10.004.

Badar, S. B., Zafar, K., Ghafoor, R. & Khan, F. R. (2022). Radiographic evaluation of the margins of clinically acceptable metal-ceramic crowns. *Journal of the Pakistan Medical Association*, *72*(Suppl 1) (2), S35-S39. https://doi.org/10.47391/JPMA.AKU-08.

Burke, F. J. T. & Lucarotti, P. S. K. (2012). Ten-year survival of bridges placed in the General Dental Services in England and Wales. *Journal of Dentistry*, *40*(11). https://doi.org/10.1016/j.jdent.2012.07.002.

Chen, L.-Y., Cui, Y.-W. & Zhang, L.-C. (2020). Recent Development in Beta Titanium Alloys for Biomedical Applications. *Metals*, *10*, 1139.

Codrea, L., Tripa, M.-S., Opruta, D., Gyorbiro, R. & Suciu, M. (2023). Transfer-Matrix Method for Calculus of Long Cylinder Tube with Industrial Applications. *Mathematics*, *11*(17), 3756. https://doi.org/10.3390/math11173756.

Cojocariu-Oltean, O., Tripa, M.-S., Baraian, I., Rotaru, D.-I. & Suciu, M. (2024). About Calculus Through the Transfer Matrix Method of a Beam with Intermediate Support with Applications in Dental Restorations. *Mathematics*, *12*(23), 3861. https://doi.org/10.3390/math12233861.

Dikova, T. (2018). Bending fracture of Co-Cr dental bridges, produced by additive technologies: experimental investigation. *Procedia Structural Integrity*, *13*, 461-468. https://doi.org/10.1016/j.prostr.2018.12.077.

Dikova, T., Dzhendov, D., Ivanov, D. & Bliznakova, K. (2018). Dimensional accuracy and surface roughness of polymeric dental bridges produced by different 3D printing processes. *Archives of Materials Science and Engineering*, *94*(2). 65-75.

Dikova, T. & Vasilev, T. (2019). Bending fracture of Co-Cr dental bridges, produced by additive technologies: Simulation analysis and test. *Engineering Fracture Mechanics*, 218/2019. https://doi.org/10.1016/j.engfracmech.2019.106583.

Gaspar, M. & Weichert, F. (2023). Integrated construction and simulation of tool paths for milling dental crowns and bridges. *Computer-Aided Design*, *45*/2013, 1170–1181.

Gery, P.-M. & Calgaro, J.-A. (1987). *Les Matrices-Transfert dans le calcul des structures*. Editions Eyrolles, Paris, France.

Ifwandi, I. (2023). Dental Bridge Procedure to Straighten Loose Teeth, A Review. *Journal of Syiah Kuala Dentistry Society*, *8*(1), 76-83. https://doi.org/10.24815/jds. v8i1.33361.

Jovanovic, M., Živic, M. & Milosavljevic, M. (2021). A potential application of materials based on a polymer and CAD/CAM composite resins in prosthetic dentistry. *Journal of Prosthodontic Research*, *65*(2), 137-47.

Lu, Z., Tao, D. & Quian, L. (2021). Buckling of Piles in Layered Soils by Transfer-Matrix Method. *International Journal of Structural Stability and Dynamics*, *21*(8), 2150109. https://doi.org/10.1142/S0219455421501091.

Mekled, S., Iskander, M., Rodriguez, B., Hodges, P., Bhogal, J., Adechoubou, J. & Weinstein, G. (2024). Fracture resistance of CAD/CAM provisional crowns with two different designs: an in vitro study. *Explor. Med.*, *5*, 566-573. https://doi.org/10.37349/emed.2024.00240.

Narwani, S., Yadav, N. S., Hazari, P. et al. (2022). Comparison of Tensile Bond Strength of Fixed-Fixed Versus Cantilever Single-and Double-Abutted Resin-Bonded Bridges Dental Prosthesis. *Materials*, *15*(16), 5744.

Nejat, A. H., Dupree, P., Kee, E., Xu, X., Zakkour, W., Odom, M., Bruggers, K. & Mascarenhas, F. (2021). Effect of Endodontic Access Preparation on Fracture Load of Translucent versus Conventional Zirconia Crowns with Varying Occlusal Thickness. *J Prosthodont.*, *30*(8), 706-710. https://doi.org/10.1111/jopr.13337.

Niinom, M. & Nakai, M. (2011). Titanium-Based Biomaterials for Preventing Stress Shielding between Implant Devices and Bone. *Int. J. Biomater*., *2011*, 836587.

Pereira, G. K. R., Guilardi, L. F., Dapieve, K. S., Kleverlaan, C. J., Rippe, M. & Valandro, L. F. (2018). Mechanical reliability, fatigue strength and survival analysis of new polycrystalline translucent zirconia ceramics for monolithic restorations. *J. Mech. Beh. Biomed. Mater*., *85*, 57–65.

Quinna, G. D., Studart, A. R., Hebert, C., Verhoef, J. R. & Arola, D. (2010). Fatigue of zirconia and dental bridge geometry: Design implications. *Dental Materials*, *26*/2010, 1133–1136. https://doi.org/10.1016/j.dental.2010.07.014.

Rezaie, F., Farshbaf, M., Dahri, M., Masjedi, M., Maleki, R., Amini, F., Wirth, J., Moharamzadeh, K., Weber, F. E. & Tayebi, L. (2023). 3D Printing of Dental Prostheses: Current and Emerging Applications. *J. Compos. Sci.*, *7*, 80. https://doi.org/10.3390/jcs7020080.

Schriwer, C., Gjerdet, N. R., Arola, D. & Øilo, M. (2021). The effect of preparation taper on the resistance to fracture of monolithic zirconia crowns. *Dental Materials*, *37*(8), e427-e434. https://doi.org/10.1016/j.dental.2021.03.012.

Skjold, A, Schriwer, C. & Øilo, M. (2019). Effect of margin design on fracture load of zirconia crowns. *Eur J Oral Sci*., *127*(1), 89-96. https://doi.org/10.1111/eos.12593.

Studart, A. R., Filser, F., Kocher, P. & Gauckler, L. J. (2007). Fatigue of zirconia under cyclic loading in water and its implications for the design of dental bridges. *Dental Materials*, *23*/2007, 106–114. https://doi.org/10.1016/j.dental.2005.12.008.

Suciu, M. (2023). About an approach by Transfer-Matrix Method (TMM) for mandible body bone calculus. *Mathematics*, *11*(2), 450. https://doi.org/10.3390/math11173756.

Suciu, M. & Tripa, M.-S. (2021). Strength of Materials. *UT Prerss* Cluj-Napoca, Romania.

Sun, J.-p. & Li, Q.-n. (2011). Precise Transfer-Matrix Method for elastic-buckling analysis of compression bar. *[J] Engineering Mechanics*, *28*(7), 26-030.

Uporabo, B. (2017). A review of the surface modifications of titanium alloys for biomedical applications. *Mater. Tehnol.*, *51*, 181–193.

Uriciuc, W. A., Vermesan, H. & Botean, A. I. (2019). Research on the surface of the dental alloys with cobalt-crom base. *Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering*, *62*(3).

Warren, C. Y. (1989). *ROARC'S-Formulas for Stress & Strain* (6-ème Edition). McGraw-Hill Book Company, New York, NY, SUA.