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# Calculus of Long Rectangle Plate Articulated on Both Long Sides Charged with a Linear Load Uniformly Distributed on a Line Parallel to the Long Borders through the Transfer-Matrix Method

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**Abstract**. The Transfer-Matrix Method is very special and interesting for a lot of industrial fields. This paper presents a study for an application of rectangular long plate articulated at the two long borders and charged with an uniform linear load, that act on a line parallel to the long sides. The long rectangular plate is discretized in unit beams, a beam has width equal to the unit, thickness equal to the plate thickness and length equal to the plate width. The study is made for the unit beam in two cases: the first case, in which the load acts in a certain section $x_0$ and the second case, a particularization of the first case, that is: the concentrated vertical force acts in the middle of the beam opening. All the elements for all state vectors for all the sections of the beam can be calculated and the stresses and the deformations in all sections of the beam too. By extension, the stresses and strains are calculated for the long rectangular plate articulated on both long borders. This work is original and very interesting for a lot of industrial fields.

**Key words**: long rectangle plate, unit beam, charge density, Dirac's and Heaviside's functions and operators, state vector, Transfer-Matrix

## Introduction

Plate calculus is very important in many fields, especially industrial, automotive and construction. Approach through Transfer-Matrix Method (TMM) is interesting and appropriate for iterative calculus, that is necessaire for solving shape optimization problems. This work presents an original calculus for a long rectangular plate articulated at two long borders and charged with a linear load uniformly distributed on a line parallel to the long borders through the TMM.

## Literature Review

Many works can be found in the field. It can be found about the calculus of plates by the series representation of the deflection function (Ahanova, Yessenbayeva & Tursyngaliev, 2016) and about an analysis of homogeneous and non-homogeneous plates (Altenbach, 2009) and a review of a few selected theories of plates in bending too (Vijayakumar, 2014). It is presenting the theory of plates and shells (Bhavikatti, 2024), the classical theory of plates (Volokh, 1994) and an analysis of the hypotheses used when constructing the theory of beams and plates (Zveryayev, 2003). The basis of TMM is presented (Gery & Calgaro, 1987). About the TMM applied in different fields can be shown: calculus of long cylinder tube with industrial applications (Codrea et al., 2023); calculus of a beam with intermediate support with applications in dental restorations (Cojocariu-Oltean et al., 2024); buckling of piles in layered soils by TMM (Lu, Tao & Qijian, 2021); about an approach by TMM for mandible body bone calculus (Suciu, 2023). Some theoretical and experimental extensions based on the properties of the intrinsic Transfer Matrix are given (Cretu, Pop & Andia Prado, 2022). A solution of thin rectangular plates with various boundary conditions it can be found (Delyavskyy et al., 2023), calculation of plate-beam systems by method of boundary elements (Surianinov, & Shyliaiev, 2018) and a determination of the stress-strain state in thin orthotropic plates on Winkler's elastic foundations too (Delyavs'kyi et al., 2015). It is presented the equivalent systems for the analysis of rectangular plates of varying thickness (Fertis & Lee, 1990). Theoretical and comparative study regarding the mechanical response under the static loading for different rectangular plates can be found (Fetea, 2018) and stress–strain analysis of rectangular plates with a variable thickness and constant weight is given (Grigorenko & Rozhok, 2002). An exact solution for the deflection of a clamped rectangular plate under uniform load is given (Imrak & Gerdemeli, 2007), a study about stress state of compound polygonal plate is presented (Kuliyev, 2003) and stress-strain state in the corner points of a clamped plate under uniformly distributed normal load (Matrosov & Suratov, 2018). About some aspects of implementation of boundary elements method in plate theory is given (Kutsenko, Kutsenko & Yaremenko, 2021) and analysis of folded plate structures by a combined boundary element-transfer matrix method too (Ohga, Shigematsu & Kohigashi, 1991). About the bending of clamped rectangular plates is presented (Meleshko & Gomilko, 1994) and about static analysis of an orthotropic plate too (Moubayed et al., 2014). It is given about application of numerical methods in solving a phenomenon of the theory of thin plates (Nikoloc Stanojevic, Dolicanin & Radojkovic, 2010) and the convergence analysis of finite element approach to classical approach for analysis of plates in bending too (Niyonyungu & Karangwa, 2019). Method of matched sections as a beam-like approach for plate analysis is shown (Orynyak & Danylenko, 2024) and an elastic analysis and application tables of rectangular plates with unilateral contact support conditions too (Papanikolaou & Doudoumis, 2001). About the development of two-dimensional theory of thick plates bending on the basis of general solution of Lamé equations is presented (Revenko, 2018) and about the determination of plane stress-strain states of the plates on the basis of the three-dimensional Theory of Elasticity too (Revenko & Revenko, 2017). Analytical solutions of the mechanical answer of thin orthotropic plates are given (Sprintu & Fuiorea, 2013). Classical calculus of Resistance of Materials is given (Suciu & Tripa, 2021; Warren, 1989). A precise TMM for elastic-buckling analysis of compression bar can be found (Sun & Li, 2011), on the calculation of the rectangular finite element of the plate (Yessenbayeva, Yesbayeva & Makazhanova, 2018) and on the finite element method for calculation of rectangular plates too (Yessenbayeva, Yesbayeva & Syzdykova, 2019). It can be found the bending of rectangular plate with each edges arbitrary point supported under a concentrated load (Yuhong, 2000).

## Methods

### Analytical calculus for a long rectangular plate

The study has begun with the analytical calculus for a long rectangular plate, (Gery & Calgaro, 1987), that is considered charged to along a cylindrical surface, request obtained as a result of a load with a charge density *q(x)* acting along of long border (a generator), as in Figure 1., a.

### Working assumptions

It is considered the plate to be subject to an axial force along the long sides, per unit side being *F* (as in Figure 1., b.).

*q(x)*, the charge density is considered to act per unit of length.

The bending moment due to a single external load is denoted by *m(x)*.

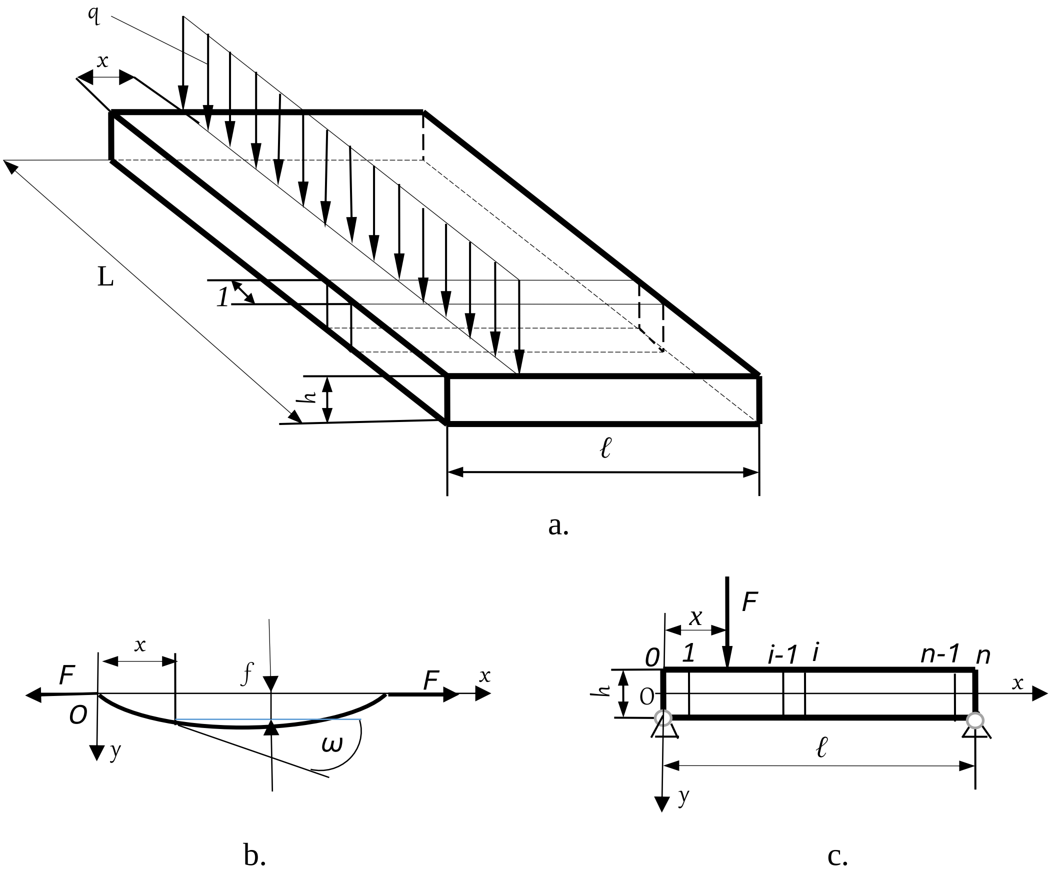



**Figure 1**. **Long rectangular plate and its discretization: a. a long rectangle plate discretized into beams (pieces) with width equal at *unity*; b. the plate to be subject to an axial force along the long sides, per unit side being *F;* c. a unit beam (unit piece) discretized into *n* parts with *n+1* sides, the unit beam is articulated at both ends**

**Calculus of the arrow for the unit beam**

It is considered that the plate is discretized in unit beams, as in Figure 1., a. The unit beam has the length *l*, equal to the width of the long rectangular plate. The unit beam is articulated at both ends as in Figure 1., c.

The total moment *M(x)* is as (1):

$$M(x) = m(x) + Ff(x) \tag{1}$$

when *f(x)* is the arrow corresponding to the section *x* and with (2):

$$\frac{d^2m}{dx^2} = q(x) \tag{2}$$

The differential equation of the average deformed fiber can be written as (3):

$$\frac{d^4m}{dx^4} - \frac{12(1-\nu^2)}{Eh^3} \cdot \frac{d^2f(x)}{dx^2}F = \frac{12(1-\nu^2)}{Eh^3}q(x) \tag{3}$$

with *A* as the bending stiffness of a plate with the thickness *h* as (4):

$$A = \frac{Eh^3}{12(1-\nu^2)} \tag{4}$$

when: *E* is the Young's modulus; *h* is the thickness of the plate and of the beam too; *ν* is the Poisson's coefficient.

With (4), expression (3) becomes like (5):

$$\frac{d^4m}{dx^4} - \frac{1}{A} \cdot \frac{d^2f(x)}{dx^2}F = \frac{12(1-\nu^2)}{Eh^3}q(x) \tag{5}$$

It is denoted by (6):

$$\alpha^2 = \frac{1}{A} \cdot F \tag{6}$$

With the notation (6), the differential equation (5) becomes as (7):

$$\frac{d^4m}{dx^4} - \alpha^2 \frac{d^2 f(x)}{dx^2} = \frac{1}{A} q(x) \tag{7}$$

The solution of the differential equation (7) without the second term is as (8):

$$f_1(x) = B_1\, ch\, \alpha x + B_2\, sh\, \alpha x + B_3 x + B_4 \tag{8}$$

The particular solution is (9):

$$f^*(x) = \frac{1}{A\alpha^3} \int_0^x [sh\alpha(x-t) - \alpha(x-t)] q(t) dt \tag{9}$$

and must verify the conditions (10):

$$f^*(0) = f^{*\prime}(0) = f^{*\prime\prime}(0) = f^{*\prime\prime\prime}(0) \tag{10}$$

In the origin, for $x = 0$, it can be written (11):

$$\begin{cases} f(0) = f_0 \\ \omega(0) = \omega_0 \\ M(0) = M_0 \\ T(0) = T_0 \end{cases} \tag{11}$$

which can be written in terms of the integration constants $B_i$, $i = 1{,}4$ as (12):

$$\begin{cases} f_0 = B_1 + B_4 \\ \omega_0 = \alpha B_2 + B_3 \\ M_0 = \alpha^2 B_1 B_4 \\ T_0 = -\alpha^3 B_2 B_4 \end{cases} \tag{12}$$

Depending on the efforts and deformations from the origin, the four integration constants $B_i$, $i = 1{,}4$ are, (13):

$$\begin{cases} B_1 = \frac{1}{\alpha^2 A} M_0 \\ B_2 = -\frac{1}{\alpha^3 A} T_0 \\ B_3 = \omega_0 + \frac{1}{\alpha^3 A} T_0 \\ B_4 = f_0 - \frac{1}{\alpha^2 A} M_0 \end{cases} \tag{13}$$

So, the mathematical expression of the deformation is as (14):

$$f(x) = f + \omega_0 x + \frac{ch\alpha x - 1}{\alpha^2 A} M_0 + \frac{\alpha x - sh\alpha x}{\alpha^3 A} T_0 + \frac{1}{\alpha^3 A} \int_0^x [sh\alpha(x-t) - \alpha(x-t)] q(t) dt \tag{14}$$

**Premises for the calculus through the TMM for the long rectangular plates**

A rectangular plate with long lengths can be calculated through the TMM. The plate is discretized, of its lengths, into unit beams, as in Figure 1., a. and c.

Each beam has a width equal to *a unit*.

**The state vectors and the Transfer-Matrix for a long rectangular plate**

For using TMM, it can be defined the state vectors and the Transfer-Matrix.

**The state vectors**

The piece equal to *unity* is as a beam, as in Figure 1., a. and c., with a Transfer-Matrix associate and deformed, as in Figure 1., b.

The beam can also be discretized along its length (which is actually the width of the rectangular plate) into *n* parts (one beam is as in Figure 1., c.), the sectioning of the beam being perpendicular to the *Ox* axis. Each part has a left side and a right side. The first part of the beam, the left part, has the left side noted *0* and the right side is noted *1*. The last side of the last part *n* on the right of the beam, is noted by *n*, the beam having *n* parts and *n+1* sides.

A ***S**tate **V**ector* $\{SV(x)\} = \{SV\}_x$ consisting of four elements ($f(x), \omega(x), M(x), T(x)$) is associated for some side $x$, (the state vector and its elements having always the index of the side it is on), (Figure 1., c.), as (15):

$$\{SV(x)\} = \{SV\}_x = \{f(x), \omega(x), M(x), T(x)\}^{-1} = \{f_x, \omega_x, M_x, T_x\}^{-1} \quad (15)$$

when:

- $\{SV(x)\} = \{SV\}_x$ is the **S**tate **V**ector corresponding at the side $x$;
- $f(x) = f_x$ is the arrow in the section $x$;
- $\omega(x) = \omega_x$ is the rotation of the average fiber in the $x$ section;
- $M(x) = M_x$ is the bending **m**oment;
- $T(x) = T_x$ is the cutting force at the $x$-axis point.

For the side $0$, for $x = 0$, it can be written (16):

$$\{SV(0)\} = \{SV\}_0 = \{f(0), \omega(0), M(0), T(0)\}^{-1} = \{f_0, \omega_0, M_0, T_0\}^{-1} \quad (16)$$

For the last part $n$, the right part (the right end of the beam), for the last side $n$, for which $x = l$, the state vector can be written as (17):

$$\{SV(l)\} = \{SV\}_l = \{f(l), \omega(l), M(l), T(l)\}^{-1} = \{f_l, \omega_l, M_l, T_l\}^{-1} \quad (17)$$

**The Transfer-Matrix**

A **T**ransfer-**M**atrix $[TM]_x$ is associated for some part $x$ of the beam, that connects two consecutive sides of a part of the beam according to the following matrix relation for the part *1*, between the sides *0* and *1* as (18):

$$\{SV\}_1 = [TM']_1\{SV\}_0 + \{SVe\}_1 \quad (18)$$

when:

- $\{SVe\}_1$ is the **S**tate **V**ector corresponding to the ***e**xternal* forces on part *1*.

For some part $x$, the matrix relation (18) can be written as (19):

$$\{SV\}_x = [TM]_x\{SV\}_0 + \{SVe\}_x \quad (19)$$

The state vector at the origin is calculated after the conditions of the ends of the unit beam. After, the general formula for the deformations gives the deformations in all sections of the rectangular plate.

**The Transfer-Matrix for a long rectangular plate**

The matrix relation (19), in the case of a long rectangular plate, with the mathematical formulation given by Dirac's and Heaviside's functions and operators (Gery & Calgaro, 1987), can be written as (20):

$$\begin{Bmatrix} f(x) \\ \omega(x) \\ M(x) \\ Y(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{ch\alpha x-1}{\alpha^2 A} & \frac{\alpha x - sh\alpha x}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha x}{\alpha A} & -\frac{ch\alpha x-1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha x}{A} & -\frac{sh\alpha x}{\alpha A} \\ 0 & 0 & \frac{\alpha h\alpha x}{A} & -\frac{ch\alpha x}{A} \end{bmatrix} \begin{Bmatrix} f_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} + \frac{F}{A} \begin{Bmatrix} \frac{1}{\alpha^3}\int_0^x [sh\alpha(x-t) - \alpha(x-t)]q(t)dt \\ \frac{1}{\alpha^2}\int_0^x [ch\alpha(x-t) - 1]q(t)dt \\ \frac{1}{\alpha}\int_0^x sh\alpha(x-t)\, q(t)dt \\ \int_0^x ch\alpha(x-t)\, q(t)dt \end{Bmatrix} \quad (20)$$

The matrix relation (20) can be written simplified as (21):

$$\{SV(x)\} = [TM]_x\{SV\}_0 + \frac{1}{A}\{SVe\}_x \quad (21)$$

when the Transfer-Matrix $[TM]_x$ is (22):

$$[TM]_x = \begin{bmatrix} 1 & x & \frac{ch\alpha x-1}{\alpha^2 A} & \frac{\alpha x - sh\alpha x}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha x}{\alpha A} & -\frac{ch\alpha x-1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha x}{A} & -\frac{sh\alpha x}{\alpha A} \\ 0 & 0 & \frac{\alpha h\alpha x}{A} & -\frac{ch\alpha x}{A} \end{bmatrix} \quad (22)$$

## Results and Discussion

**Applications for the calculus of a long rectangular plate articulated at the two long borders charged with a linear load uniformly distributed on a line parallel to the long borders through the TMM**

The matrix relation (19) and the expressions (15), (16) and (17) serve to calculate the efforts and deformations in the origin section, and then, the efforts, deformations and stresses can be calculated in any section of the beam, respectively for the long rectangular plate (Gery & Calgaro, 1987).

In the following, two cases of loading with a concentrated vertical force will be studied for the plate articulated on the two long borders. It will study beams with width equal to *unity* (which were obtained by discretizing the long rectangular plate, according to Figure 1., a.) with the length equal to the width *l* of the plate and with the thickness *h*, equal to the thickness of the long rectangular plate.

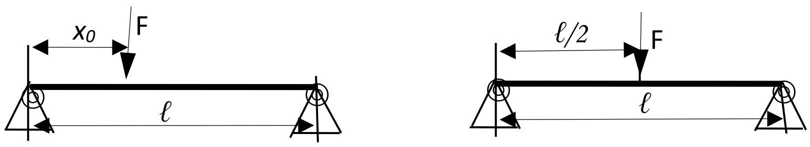
**Unit width beam articulated at the two ends charged with concentrated load (Figure 2., a.) at the section $x_0$**

It is considered a unit beam articulated at the two edges, charged with a concentrated load *F* in some section $x_0$ (Figure 2., a.). The charge density can be written as (23):

$$q(x) = -F(x - x_0) \tag{23}$$

With Dirac's and Heaviside's functions and operators, the state vector $\{SVe\}_x$ corresponding at exterior load is as (24):

$$\{SVe\}_x = \begin{Bmatrix} -\frac{1}{\alpha^3 A} FY(x - x_0)[sh\alpha(x - x_0) - \alpha(x - x_0)] \\ -\frac{1}{\alpha^2 A} FY(x - x_0)[ch\alpha(x - x_0) - 1] \\ -\frac{1}{\alpha A} FY(x - x_0)\, sh\alpha(x - x_0) \\ -\frac{1}{A} FY(x - x_0)\, ch\alpha(x - x_0) \end{Bmatrix} \tag{24}$$




a. b.

**Figure 2. Unitary beam articulated at both ends with a concentrated vertical load *F*: a. concentrated vertical load *F* which acts in a certain section $x_0$; b. concentrated vertical load *F* which acts in a section at $x_0 = l/2$.**

Matrix relation, in this case is as (25):

$$\begin{Bmatrix} f(x) \\ \omega(x) \\ M(x) \\ Y(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{ch\alpha x - 1}{\alpha^2 A} & \frac{\alpha x - sh\alpha x}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha x}{\alpha A} & -\frac{ch\alpha x - 1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha x}{A} & -\frac{sh\alpha x}{\alpha A} \\ 0 & 0 & \frac{\alpha h\alpha x}{A} & -\frac{ch\alpha x}{A} \end{bmatrix} \begin{Bmatrix} f_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} - \frac{1}{A} \begin{Bmatrix} \frac{1}{\alpha^3} FY(x - x_0)[sh\alpha(x - x_0) - \alpha(x - x_0)] \\ \frac{1}{\alpha^2} FY(x - x_0)[ch\alpha(x - x_0) - 1] \\ \frac{1}{\alpha} FY(x - x_0) sh\alpha(x - x_0) \\ FY(x - x_0) ch\alpha(x - x_0) \end{Bmatrix} \tag{25}$$

With Dirac's and Heaviside's functions and operators the matrix relation (25) can be written as (26):

$$\begin{Bmatrix} f(x) \\ \omega(x) \\ M(x) \\ Y(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{ch\alpha x-1}{\alpha^2 A} & \frac{\alpha x - sh\alpha x}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha x}{\alpha A} & -\frac{ch\alpha x-1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha x}{A} & -\frac{sh\alpha x}{\alpha A} \\ 0 & 0 & \frac{\alpha h\alpha x}{A} & -\frac{ch\alpha x}{A} \end{bmatrix} \begin{Bmatrix} f_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} - \frac{1}{A} \begin{Bmatrix} \frac{1}{\alpha^3}[Fsh\alpha(x-x_0) - \alpha(x-x_0)] \\ \frac{1}{\alpha^2}F[ch\alpha(x-x_0)-1] \\ \frac{1}{\alpha}Fsh\alpha(x-x_0) \\ Fch\alpha(x-x_0) \end{Bmatrix} \tag{26}$$

For the right end, for $x = l$, expression (26) becomes (27):

$$\begin{Bmatrix} f(l) \\ \omega(l) \\ M(l) \\ T(l) \end{Bmatrix} = \begin{bmatrix} 1 & l & \frac{ch\alpha l-1}{\alpha^2 A} & \frac{\alpha - sh\alpha l}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha l}{\alpha A} & -\frac{ch\alpha l-1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha l}{A} & -\frac{sh\alpha l}{\alpha A} \\ 0 & 0 & \frac{\alpha sh\alpha l}{A} & -\frac{ch\alpha l}{A} \end{bmatrix} \begin{Bmatrix} f_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} - \begin{Bmatrix} \frac{1}{\alpha^3 A}F[sh\alpha(l-x_0) - \alpha(l-x_0)] \\ \frac{1}{\alpha^2 A}F[ch\alpha(l-x_0)-1] \\ \frac{1}{\alpha A}F\, sh\alpha(l-x_0) \\ \frac{1}{A}F\, ch\alpha(l-x_0) \end{Bmatrix} \tag{27}$$

The conditions for the two articulated ends of the beam are (28):

$$\begin{cases} f_0 = 0 \\ M_0 = 0 \\ f(l) = 0 \\ M(l) = 0 \end{cases} \tag{28}$$

With (28), the matrix relation (27) becomes (29):

$$\begin{Bmatrix} 0 \\ \omega(l) \\ 0 \\ T(l) \end{Bmatrix} = \begin{bmatrix} 1 & l & \frac{ch\alpha l-1}{\alpha^2 A} & \frac{\alpha - sh\alpha l}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha l}{\alpha A} & -\frac{ch\alpha l-1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha l}{A} & -\frac{sh\alpha l}{\alpha A} \\ 0 & 0 & \frac{\alpha sh\alpha l}{A} & -\frac{ch\alpha l}{A} \end{bmatrix} \begin{Bmatrix} 0 \\ \omega_0 \\ 0 \\ T_0 \end{Bmatrix} + \begin{Bmatrix} -\frac{1}{\alpha^3 A}F[sh\alpha(l-x_0) - \alpha(l-x_0)] \\ -\frac{1}{\alpha^2 A}F[ch\alpha(l-x_0)-1] \\ -\frac{1}{\alpha A}F\, sh\alpha(l-x_0) \\ -\frac{1}{A}F\, ch\alpha(l-x_0) \end{Bmatrix} \tag{29}$$

or, it can be written more simply as (30):

$$\begin{Bmatrix} 0 \\ \omega(l) \\ 0 \\ T(l) \end{Bmatrix} = \begin{bmatrix} tm_{11} & tm_{12} & tm_{13} & tm_{14} \\ tm_{21} & tm_{22} & tm_{23} & tm_{24} \\ tm_{31} & tm_{23} & tm_{33} & tm_{34} \\ tm_{41} & tm_{42} & tm_{43} & tm_{44} \end{bmatrix} \begin{Bmatrix} 0 \\ \omega_0 \\ 0 \\ T_0 \end{Bmatrix} + F \begin{Bmatrix} sv_{e_1} \\ sv_{e_2} \\ sv_{e_3} \\ sv_{e_4} \end{Bmatrix} \tag{30}$$

By writing the first lines and the third line from the matrix relation (30), a linear system of two equations with two unknowns is obtained as (31), the unknowns being the two elements of the state vector of side *0*, $\omega_0$ and $T_0$:

$$\begin{cases} tm_{12}\omega_0 + tm_{14}T_0 = -sv_{e_1}F \\ tm_{32}\omega_0 + tm_{34}T_0 = -sv_{e_3}F \end{cases} \tag{31}$$

with solution (32):

$$\begin{cases} \omega_0 = \frac{sv_{e_1}tm_{34} - sv_{e_3}tm_{14}}{tm_{14}tm_{32} - tm_{12}tm_{31}}F = c_1F \\ T_0 = \frac{sv_{e_1}tm_{32} - sv_{e_3}tm_{12}}{tm_{12}tm_{34} - tm_{14}tm_{32}}F = c_2F \end{cases} \tag{32}$$

With (332), the other two elements of the state vector for the last side on the right end, the side *n*, for $x = l$, will be like (33):

$$\begin{cases} \omega(l) = tm_{22}\omega_0 + tm_{24}T_0 + sv_{e_2}F \\ T(l) = tm_{42}\omega_0 + tm_{44}T_0 + sv_{e_4}F \end{cases} \tag{33}$$

Now, all elements of any state vector of any side can be calculated for all parts in which the unit beam has been discretized.

**Unit width beam articulated at the two long borders charged with a concentrated vertical load, (Figure 2., b.), at the middle, for $x_0 = l/2$**

For the vertical concentrated load at the middle, for $x_0 = l/2$, (Figure 2., b.), the charge density is as (34):

$$q'(x) = -\mathrm{F}\left(x - \frac{l}{2}\right) \tag{34}$$

If the load is at the middle of the unit beam, respectively at the middle of the width of the long rectangular plate, for $x_0 = l/2$, the matrix relation (26) can be written as (35):

$$\begin{Bmatrix} f'(x) \\ \omega'(x) \\ M'(x) \\ T'(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{ch\alpha x - 1}{\alpha^2 A} & \frac{\alpha x - sh\alpha x}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha x}{\alpha A} & -\frac{ch\alpha x - 1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha x}{A} & -\frac{sh\alpha x}{\alpha A} \\ 0 & 0 & \frac{\alpha sh\alpha x}{A} & -\frac{ch\alpha x}{A} \end{bmatrix} \begin{Bmatrix} f'_0 \\ \omega'_0 \\ M'_0 \\ T'_0 \end{Bmatrix} + F \begin{Bmatrix} -\frac{1}{\alpha^3 A}\left[sh\alpha\left(x - \frac{l}{2}\right) - \alpha\left(x - \frac{l}{2}\right)\right] \\ -\frac{1}{\alpha^2 A}\left[ch\alpha\left(x - \frac{l}{2}\right) - 1\right] \\ -\frac{1}{\alpha A}\, sh\alpha\left(x - \frac{l}{2}\right) \\ -\frac{1}{A}\, ch\alpha\left(x - \frac{l}{2}\right) \end{Bmatrix} \tag{35}$$

In the right extremity of the beam, for $x = l$, the matrix relation (35) can be written as (36):

$$\begin{Bmatrix} f'(l) \\ \omega'(l) \\ M'(l) \\ T'(l) \end{Bmatrix} = \begin{bmatrix} 1 & l & \frac{ch\alpha l - 1}{\alpha^2 A} & \frac{\alpha l - sh\alpha l}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha l}{\alpha A} & -\frac{ch\alpha l - 1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha l}{A} & -\frac{sh\alpha l}{\alpha A} \\ 0 & 0 & \frac{\alpha sh\alpha l}{A} & -\frac{ch\alpha l}{A} \end{bmatrix} \begin{Bmatrix} f'_0 \\ \omega'_0 \\ M'_0 \\ T'_0 \end{Bmatrix} + F \begin{Bmatrix} -\frac{1}{\alpha^3 A}\left[sh\alpha\left(l - \frac{l}{2}\right) - \alpha\left(l - \frac{l}{2}\right)\right] \\ -\frac{1}{\alpha^2 A}\left[ch\alpha\left(l - \frac{l}{2}\right) - 1\right] \\ -\frac{1}{\alpha A}\, sh\alpha\left(l - \frac{l}{2}\right) \\ -\frac{1}{A}\, ch\alpha\left(l - \frac{l}{2}\right) \end{Bmatrix} \tag{36}$$

or, (37):

$$\begin{Bmatrix} f'(l) \\ \omega'(l) \\ M'(l) \\ T'(l) \end{Bmatrix} = \begin{bmatrix} 1 & l & \frac{ch\alpha l - 1}{\alpha^2 A} & \frac{\alpha l - sh\alpha l}{\alpha^3 A} \\ 0 & 1 & \frac{sh\alpha l}{\alpha A} & -\frac{ch\alpha l - 1}{\alpha^2 A} \\ 0 & 0 & \frac{ch\alpha l}{A} & -\frac{sh\alpha l}{\alpha A} \\ 0 & 0 & \frac{\alpha sh\alpha l}{A} & -\frac{ch\alpha l}{A} \end{bmatrix} \begin{Bmatrix} f'_0 \\ \omega'_0 \\ M'_0 \\ T'_0 \end{Bmatrix} + F \begin{Bmatrix} -\frac{1}{\alpha^3 A}\left(sh\alpha\frac{l}{2} - \alpha\frac{l}{2}\right) \\ -\frac{1}{\alpha^2 A}\left(ch\alpha\frac{l}{2} - 1\right) \\ -\frac{1}{\alpha A}\, sh\alpha\frac{l}{2} \\ -\frac{1}{A}\, ch\alpha\frac{l}{2} \end{Bmatrix} \tag{37}$$

or, simplified as (38):

$$\begin{Bmatrix} f'(l) \\ \omega'(l) \\ M'(l) \\ T'(l) \end{Bmatrix} = \begin{bmatrix} tm_{11} & tm_{12} & tm_{13} & tm_{14} \\ tm_{21} & tm_{22} & tm_{23} & tm_{24} \\ tm_{31} & tm_{23} & tm_{33} & tm_{34} \\ tm_{41} & tm_{42} & tm_{43} & tm_{44} \end{bmatrix} \begin{Bmatrix} f'_0 \\ \omega'_0 \\ M'_0 \\ T'_0 \end{Bmatrix} + F \begin{Bmatrix} sv'_{e_1} \\ sv'_{e_2} \\ sv'_{e_3} \\ sv'_{e_4} \end{Bmatrix} \tag{38}$$

With conditions (29) relation (38) becomes as (39):

$$\begin{Bmatrix} 0 \\ \omega'(l) \\ 0 \\ T'(l) \end{Bmatrix} = \begin{bmatrix} tm_{11} & tm_{12} & tm_{13} & tm_{14} \\ tm_{21} & tm_{22} & tm_{23} & tm_{24} \\ tm_{31} & tm_{23} & tm_{33} & tm_{34} \\ tm_{41} & tm_{42} & tm_{43} & tm_{44} \end{bmatrix} \begin{Bmatrix} 0 \\ \omega'_0 \\ 0 \\ T'_0 \end{Bmatrix} + F \begin{Bmatrix} sv'_{e_1} \\ sv'_{e_2} \\ sv'_{e_3} \\ sv'_{e_4} \end{Bmatrix} \tag{39}$$

From the matrix relation (39), if lines one and three are written, it is obtained a linear system of two equations with two unknowns as (40), the unknowns being the other two elements of the state vector of side $0$, $\omega_0$ and $T_0$:

$$\begin{cases} tm_{12}\omega'_0 + tm_{14}T'_0 = -sv'_{e_1}F \\ tm_{32}\omega'_0 + tm_{34}T'_0 = -sv'_{e_3}F \end{cases} \tag{40}$$

with solution (41):

$$\begin{cases} \omega'_0 = \frac{sv'_{e_1}tm_{34} - sv'_{e_3}tm_{14}}{tm_{14}tm_{32} - tm_{12}tm_{31}}F = c'_1F \\ T'_0 = \frac{sv'_{e_1}tm_{32} - sv'_{e_3}tm_{12}}{tm_{12}tm_{34} - tm_{14}tm_{32}}F = c'_2F \end{cases} \tag{41}$$

From (39), the other two elements of the state vector for the last side on the right end, the side $n$, for $x = l$, will be like (42):

$$\begin{cases} \omega'(l) = tm_{22}\omega'_0 + tm_{24}T'_0 + sv'_{e_2}F \\ T'(l) = tm_{42}\omega'_0 + tm_{44}T'_0 + sv'_{e_4}F \end{cases} \tag{42}$$

Now, all elements of any state vector of any side can be calculated for all parts in which the unit beam has been discretized.

## Conclusion

TMM is a very interesting approach to solve many problems. This work presents one of the multiple applications of TMM, in the calculation of long rectangular plates, loaded with a linear load uniformly distributed on a line parallel to the long sides. The long rectangular plate is discretized into beams with unit width, with thickness equal to the thickness of the rectangular plate and with length equal to the width of the rectangular plate. It is considered that the unit beam is charged with a concentrated vertical load, in two cases: the first case, in which the load act in a certain section $x_0$ and the second case, a particularization of the first case, that is: the concentrated vertical force acts in the middle of the beam opening. It is writing the matrix relation that connects the state vector of section *0*, from the left end, with the state vector of some section $x$. After, it is writing the matrix relation between the state vector of the right section function of the state vector of the left section, the origin section. At this moment, in the matrix relation, the conditions of the extremities can be placed on the two supports at the two ends of the beam. This matrix relation can be developed in a linear system of equations from which the unknowns from the origin section can be calculated at the beginning and then, from the last section, the section on the right. After, all the elements for all state vectors for all the sections of the beam can be calculated and the stresses and the deformations in all sections of the beam too. By extension, the stresses and strains are calculated for the long rectangular plate articulated on both long borders. This work is original and very interesting for a lot of industrial fields.

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