Comparative Analysis of North West Corner (NWC) and Modified Distribution (MODI) Programs of Solving Distribution Problem

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Abstract. Most companies involved in the distribution of manufactured goods are facing optimal cost problem in the distribution of their goods from source(s) to destination(s). As a result of this, many researches have been carried out to solve the distribution problem for cost optimal solution.

This paper selected two algorithms (Northwest Corner and Modified Improvement Index) used to solve distribution problems and checked the performance of the implemented programs written in Java for the algorithms and justify the better program. It examined the better of the two Java programs for the distribution problem algorithms (Northwest Corner (NWC) and Modified Distribution (MODI)) using Coca-Cola distribution system as a case study. The metrics considered are average run time for the execution, Lines of code, solution type, and complexity of programs.

The result of the research shows that Northwest Corner method has shorter execution time (851328.4ns for NWC and 21740104.4ns for MODI) and lesser development time with shorter lines of code (LOC). NWC has 157 LOC while MODI has 408 LOC, complex algorithm and more execution time; however, the MODI program had the better optimal solution.

Keywords: Distribution Problem, Algorithms, Java Program, Northwest Corner, Modified Distribution and Line of Code

Introduction

Operations Research (OR) comprises a wide range of applications in the different areas for efficient and effective decision making in the fierce competitive business environment. It involves numerous problem solving techniques and methods for the optimal solution of the complex decision making problems (Bhavya & Arvind, 2016). Operations Research has gained wider acclaim in the modern complex business world. For every complex problem of an industry today, well defined Operations Research tools are the solace. Decision making in today's social and business environment has become a complex task (Lovely Professional University, 2012). One of the major problems in Operation Research is the Transportation or Distribution Problem.

Market distribution involves all those business activities which are necessary to move goods from the producer to the consumer and to make them available in the amounts and of the kind desired. The distribution problems are generally concerned with the distribution of a certain product from several sources to various localities at the most minimum cost. The problem assumes that the shipping cost on a given route is directly proportional to the number of units shipped on that route (Anderson, Sweeney, & Williams, 1995). Transportation problems have been widely studied in Computer Science and Operations Research. It is one of the fundamental problems of network flow problem which is usually use to minimize the transportation cost for industries with number of sources and number of destination while satisfying the supply limit and demand requirement (Hasan, 2012).

Distribution problem is one of the oldest problems when it comes to Linear Programming Problem. Transportation cost is the most important part of the total expenditure of a company along with the production cost. In generally a business company has some factories for manufacturing products and some retail centers for distributing products which are known as sources and destinations respectively in transportation model (Babu *et al.*, 2014). The transportation managers are evaluated by their decision-making. Linear programming is one of the strongest techniques which can be used by managers to solve problems considering/subject to the settings of the problem. By applying linear programming, the managers are trying to maximize their profit on one hand, and minimize their costs on the other (Lord *et al.*, 2013).

Transportation Problem is an important part of the supply chains in linear programming. The problem basically deals with the determination of a coast plan for transporting a single commodity from a number of sources to a number of destinations (Singh, Dubey, & Shrivastava, 2012).

Literature Review

The transportation models are one of these economic optimizations which have their roots in operational management and industrial mathematics as well since long back 1941 (Maurya *et al.*, 2014). In 1953, Professor P.C. Mahalanobis established an Operation Research team in the Indian Statistical Institute, Calcutta to solve problems related to national planning and survey. In 1958, project scheduling techniques and Transportations are developed as efficient tools for scheduling and monitoring lengthy, complex and expensive projects of that time (Kavitha & Vinoba, 2015).

One of the most important and successful applications of quantitative analysis to solving business problems has been in the physical distribution of products, commonly referred to as transportation problems. Basically, the purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity (Reeb & Leavengood, 2002). In the packaged goods industry, delivery trucks are empty 25% of the time. Just four years ago, Land O'Lakes truckers spent much of their time shuttling empty trucks down slow-moving highways, wasting several million dollars annually (Hong, Vaidya, & Lu, 2011).

The transportation problem can be expressed using the mathematical model and the network model. The best approach for solving the problem is the mathematical model. Mathematical model involves transportation of a single commodity from various sources of supply or origins to various demands or destinations.

Let there be m sources of supply S1, S2, …...............Sm having ai (i = 1, 2,......m) units of supplies respectively to be transported among n destinations D1, D2 ………Dn with bj (j = 1,2…..n) units of requirements respectively. Let Cij be the cost for shipping one unit of the commodity from source i, to destination j for each route. If xij represents the units shipped per route from source i, to destination j, then the problem is to determine the transportation schedule which minimizes the total transportation cost of satisfying supply and demand conditions. The transportation problem can be stated mathematically as a linear programming problem as below (Lovely Professional University, 2012):

Minimize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Notes subject to constraints;

$$\sum_{j=1}^{n} X_{ij} = a_{i,} \ldots\ldots i = 1,2, \ldots\ldots n \ (supply\ constraints)$$

$$\sum_{i=1}^{m} X_{ij} = b_{j,}\ j = 1,2 \ldots\ldots m \qquad (demand\ Constraints)$$

The Operations Research Approach

This approach comprises the following seven sequential steps: (1) Orientation, (2) Problem Definition, (3) Data Collection, (4) Model Formulation, (5) Solution, (6) Model Validation and Output Analysis, and (7) Implementation and Monitoring. Tying each of these steps together is a mechanism for continuous feedback.

Using techniques such as mathematical modeling to analyze complex situations, operations research gives executives the power to make more effective decisions and build more productive systems based on:

i. More complete data
ii. Consideration of all available options
iii. Careful predictions of outcomes and estimates of risk
iv. The latest decision tools and techniques

Algorithms of Linear Programming

A series of linear constraints on two variables produces a region of possible values for those variables. Solvable problems will have a feasible region in the shape of a simple polygon.

The Simplex Algorithm

The simplex algorithm, developed by George Dantzig, solves LP problems by constructing a feasible solution at a vertex of the polytope and then walking along a path on the edges of the polytope to vertices with non-decreasing values of the objective function until an optimum is reached. In many practical problems, "stalling" occurs: Many pivots are made with no increase in the objective function. In rare practical problems, the usual versions of the simplex algorithm may actually "cycle". To avoid cycles, researchers developed new pivoting rules.

In practice, the simplex algorithm is quite efficient and can be guaranteed to find the global optimum if certain precautions against cycling are taken. The simplex algorithm has been proved to solve "random" problems efficiently, i.e. in a cubic number of steps (Borgwadt, Todd), which is similar to its behavior on practical problems

However, the simplex algorithm has poor worst-case behavior: Klee and Minty constructed a family of linear programming problems for which the simplex method takes a number of steps exponential in the problem size. In fact, for some time it was not known whether the linear programming problem was solvable in polynomial time.

The Ellipsoid Algorithm

Ellipsoid method was introduced by Leonid Khachiyan in 1979 to resolve the long standing issue of operation research, the first worst-case polynomial-time algorithm for linear programming. To solve a problem which has n variables and can be encoded in L input bits, this algorithm uses O(n4L) pseudo-arithmetic operations on numbers with O(L) digits. Khahiyan's algorithm and his convergence analysis have (real-number) predecessors, notably the iterative methods developed by Naum Z. Shor and the approximation algorithms by Arkadi Nemirovski and D. Yudin.

Interior Point Methods

Khachiyan's algorithm was of landmark importance for establishing the polynomial-time solvability of linear programs. The algorithm had little practical impact, as the simplex method is more efficient for all but specially constructed families of linear programs. However, it inspired new lines of research in linear programming with the development of interior point methods, which can be implemented as a practical tool. In contrast to the simplex algorithm, which finds the optimal solution by progressing along points on the boundary of a polytopal set, interior point methods move through the interior of the feasible region.

Karmarkar in 1984 proposed a new method for linear programming. Karmarkar's algorithm not only improved on Khachiyan's theoretical worst-case polynomial bound (giving O(n3.5L)). Karmarkar also claimed that the algorithm exhibited practical performance improvements over the simplex method, which created great interest in interior-point methods. Many interior point methods have been proposed and analyzed after the proposal of this new algorithm. Early successful implementations were based on affine scaling variants of the method.

Related Works

There are several open problems in the theory of linear programming, the solution of which would represent fundamental breakthroughs in mathematics and potentially major advances in our ability to solve large-scale linear programs.

Singh, Dubey, and Shrivastava in 2012 worked on initial basic feasible solution. They compared Modified Improvement Index (MODI) and Zero Point (z-p) method of finding the optimal solution of Transportation problem. Their analysis was established by mean of samples among the new algorithms.

Joshi (2013) in her paper optimized three variables to reduce transportation cost using four selected methods which include: Northwest corner method, least cost method, vogel method and modi method to find the best and cheapest route on how supply will be used to satisfy demand at specific points. She concluded that MODI method employed in this paper can be used with good deal of success in solving such problems.

Patel and Bhathwala (2013) in their paper studied the advance method for the optimal solution of a transportation problem. The algorithm for proposed method discussed in their research gave an initial as well as either optimal solution or near to optimal solution. They concluded that that the proposed algorithm gives an optimal solution nearly comparable to MODI's method in less time period.

Maurya et al. (2014) reviewed some selected models for general transportation problem. They analyzed optimal solution of stochastic transportation problem.

Kavitha and Vinoba (2015) attempted to study the importance of Operation Research and various techniques used to improve the operational efficiency of the organization.

Bhavya and Arvind (2016) worked on different models of solving distribution problem. In their study, they compared these models using the Excel solver Technique for the best possible method.

Methodology

Data for this research work was collected at Coca-Cola industry at Asejire plant in Oyo state, Nigeria and used to test the developed programs for the two selected algorithms that is Northwest Corner Method and Modified Distribution Methods for comparison of transportation programs written in Java, with the view to finding the more efficient transportation programs and the better transportation network for the benefit of the transport authority. These programs calculate the execution time and the Lines of Codes (LOC).

North West Corner Method (NWC)

Start at the North-West corner, and allocate the most minimum of the supply and demand. If supply exceeds the demand, proceed to the next destination, and continue until all of supply 1 is allocated. Then, go to source 2 and repeat the allocation process, starting with the first (lowest index) destination whose demand has not been fulfilled. If demand exceeds the supply, proceed to the next source, and continue until all of demand 1 is allocated. Then, go to destination 2 and repeat the allocation process.

Procedural Steps for Northwest Corner Method

i. Begin in the upper left-hand corner of the tableau (the NW corner)
ii. Assign the largest shipment possible
   a. If the column constraint is satisfied, move to the column on the right
   b. If the row constraint is satisfied, move to the row below
iii. Continue until all row & column constraints are satisfied
iv. To find the basic feasible solution by the Northwest Corner method;

Begin in the upper left (northwest) corner of the distribution table and set x_{11} as large as possible (here the limitations for setting x_{11} to a larger number, will be the demand of demand point 1 and the supply of supply point 1. X_{11} value cannot be greater than minimum of these 2 values).

Algorithm for Northwest Corner Method

STEP 1: Start
STEP 2: Input the total number of source(s) and destination(s)
STEP 3: scount=1, dcount=1, i=1, j=1
STEP 4: Supply value into supply[scount] and demand[dcount]
STEP 5: If (SUM(supply[scount]))=(SUM(demand[dcount])) then goto 7 else goto 6
STEP 6: Display "Unbalanced distribution" then goto 2
STEP 7: Supply the cost for each distribution, C_{ij}
STEP 8: Repeat the following steps until all row and column constraints are satisfied

(i) Check the possible shipment between supply[scount] and demand[dcount]
(ii) Table[scount,dcount] = possible shipment
(iii) If (possible shipment = supply [scount]) then
 (Supply [scount] = 0
 Demand [dcount] = demand [dcount] – supply[scount]
 scount++)
 else If (possible shipment = demand [dcount]) then
  (demand[dcount] = 0
  supply [s] = supply[s] - demand[d]
  dcount++)
 else
 (demand[dcount]=0,supply[scount]=0
 Dcoount++,scount++)

STEP 9: InitialTotalCost(Tci) = table[scount][dcount] * $C_{ij}$where i = 1…s and j = 1…d
STEP 10: Display Tci
STEP 11: Stop.

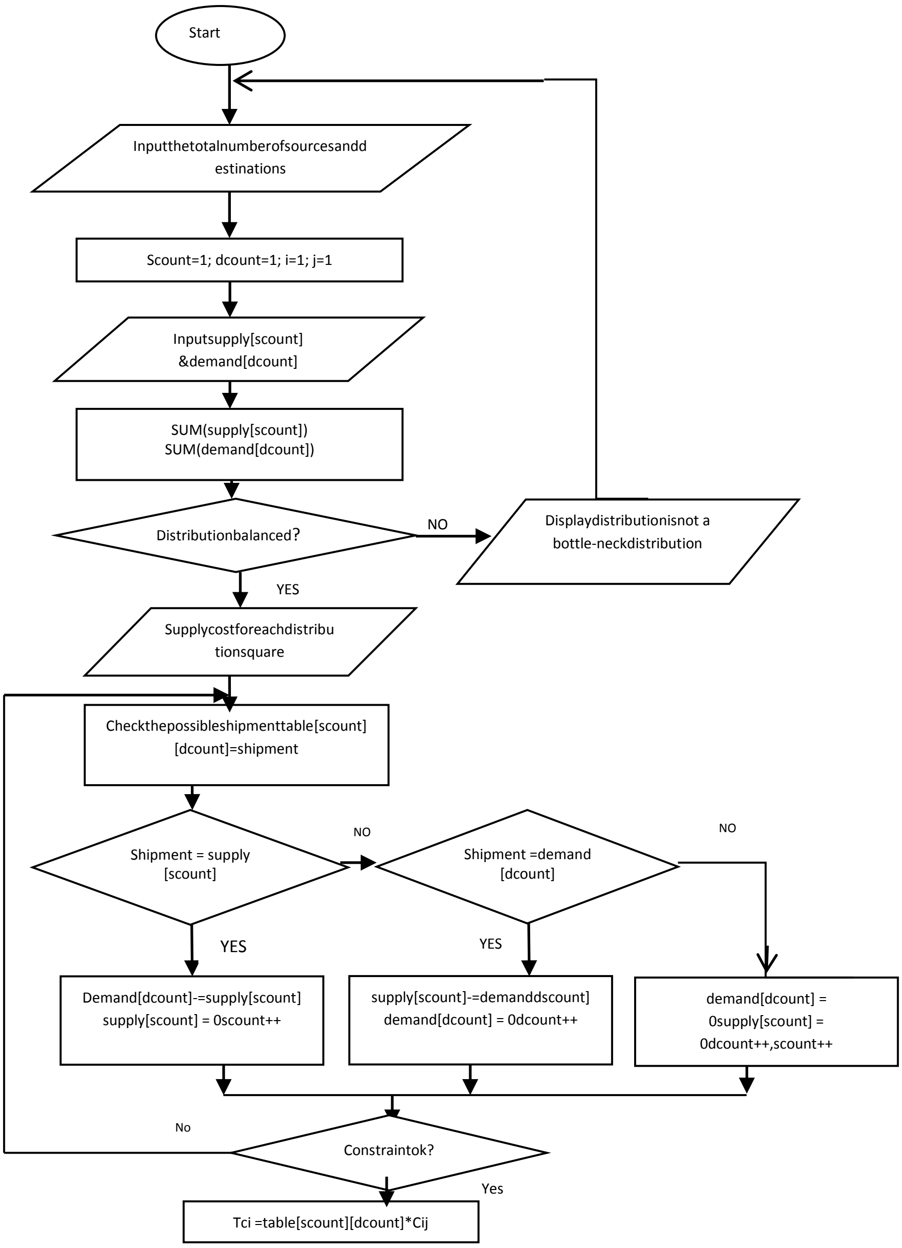


Figure 1. Flowchart for the North-West Corner Algorithm

Modified Distribution Method (MODI)

The MODI (modified distribution) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over other methods for solving distribution problems.

MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace only one closed path. This path helps determine the maximum number of units that can be shipped via the best unused route.

The MODI method then requires five steps:

i. Compute the values for each row and column, set $R_i + K_j = C_{ij}$
ii. When all equations have been written, set $R_1 = 0$.
iii. Solve the system of equations for all R and K values.
iv. Compute the improvement index for each unused square by the formula improvement index $(I_{ij}) = C_{ij} - R_i - K_j$.

v. Select the largest negative index and proceed to solve the problem as you did using the stepping-stone method.

Algorithm for Modified Distribution (MODI) Method

STEP 1: Start
STEP 2: Input the total number of source(s) and destination(s)
STEP 3: scount=1, dcount=1, i=1, j=1
STEP 4: Supply value into supply[scount] and demand[dcount]
STEP 5: If (SUM (supply [scount])) = (SUM (demand [dcount])) then goto 7 else goto 6
STEP 6: Display "Unbalanced distribution" then goto 2
STEP 7: Supply the cost for each distribution, C_{ij}
STEP 8: Repeat the following steps until all row and column constraints are satisfied

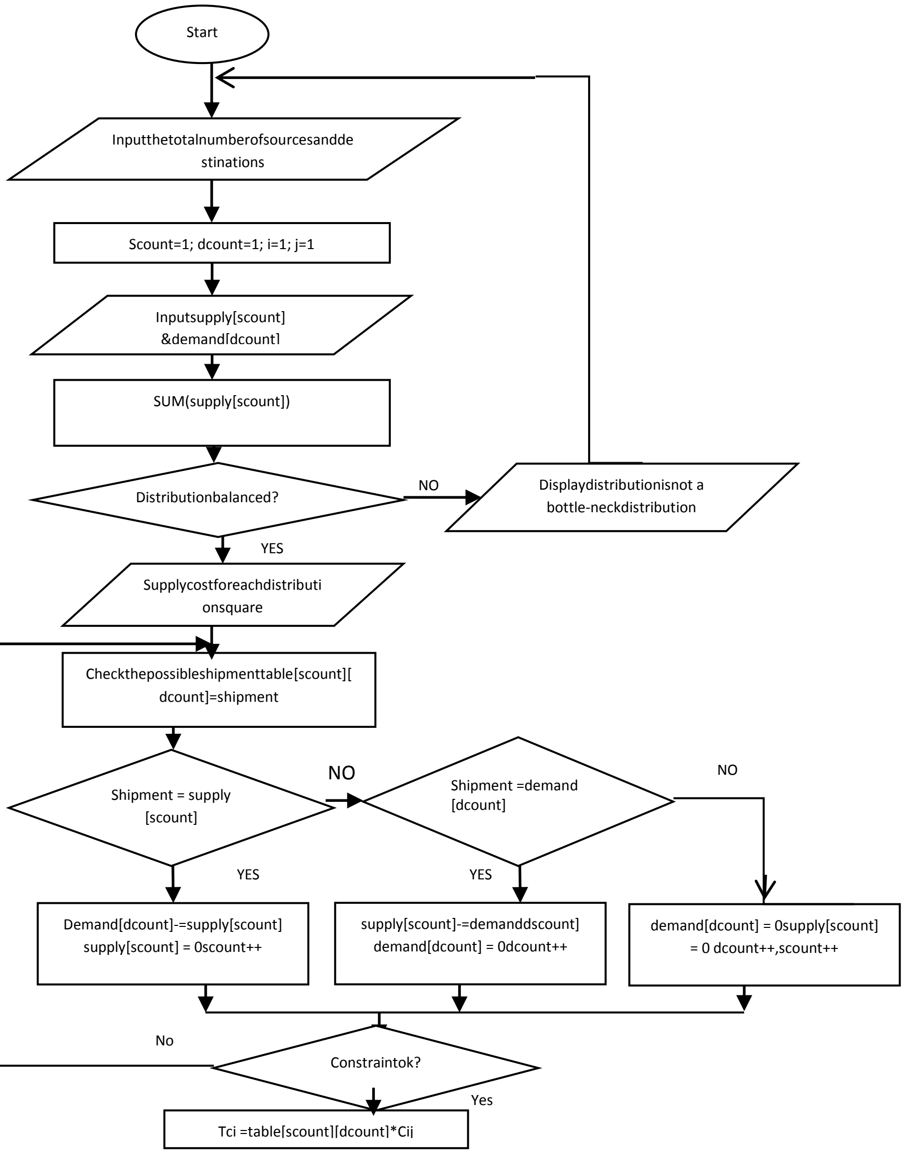
(i) Check the possible shipment between supply[scount] and demand[dcount]
(ii) Table[scount,dcount] = possible shipment
(iii) If (possible shipment = supply [scount]) then
    (Supply [scount] = 0
    Demand [dcount] = demand [dcount] – supply[scount]
    scount++)
  else If (possible shipment = demand [dcount]) then
    (demand[dcount] = 0
    supply [s] = supply[s] - demand[d]
    dcount++)
  else
    (demand[dcount]=0,supply[scount]=0
    dcoount++,scount++)

STEP 9: InitialTotalCost(Tci) = table[scount][dcount] * $C_{ij}$where i = 1…s and j = 1…d
STEP 10: Display Tci
STEP 11: $C_{ij} = R_i+K_j$for occupied space
STEP 12: $R_i = 0$
STEP 13: Improvement Index (I_{ij}) = Cij- R_i- $K_j$for unused square
STEP 14: If there is any improvement index (negative index) then goto 15 else goto 16
STEP 15: Repeat step (i) to (vi) until there is no improvement index

i. Select the largest negative index.
ii. Beginning at the square with the best improvement index, trace a closed path back to the original square through squares that are currently been used.
iii. Beginning with positive sign at the unused square, place alternate negative and positive sign on each corner square of the closed path traced in step (iii).
iv. Select smallest quantity found in those squares containing negative sign.
v. Add the number to all squares on closed path with positive sign;subtract the number from all square with negative sign.
vi. Goto step 11

STEP 16: Total cost (T_{opt}) = T_C +(Q_{ij} + C_{ij}) where i=1…total source and j=1…total destination
STEP 17: Display T_{opt} (optimal cost)
STEP 18: Stop

The flowchart of the implemented MODI algorithm is shown in Figure 2.


Start
Inputthetotalnumberofsourcesandde stinations
Scount=1; dcount=1; i=1; j=1
Inputsupply[scount] &demand[dcount]
SUM(supply[scount])
Distributionbalanced?
NO
Displaydistributionisnot a bottle-neckdistribution
YES
Supplycostforeachdistributi onsquare
Checkthepossibleshipmenttable[scount][dcount]=shipment
Shipment = supply [scount]
NO
Shipment =demand [dcount]
NO
YES
YES
Demand[dcount]-=supply[scount] supply[scount] = 0scount++
supply[scount]-=demanddscount] demand[dcount] = 0dcount++
demand[dcount] = 0supply[scount] = 0 dcount++,scount++
No
Constraintok?
Yes
Tci =table[scount][dcount]*Cij

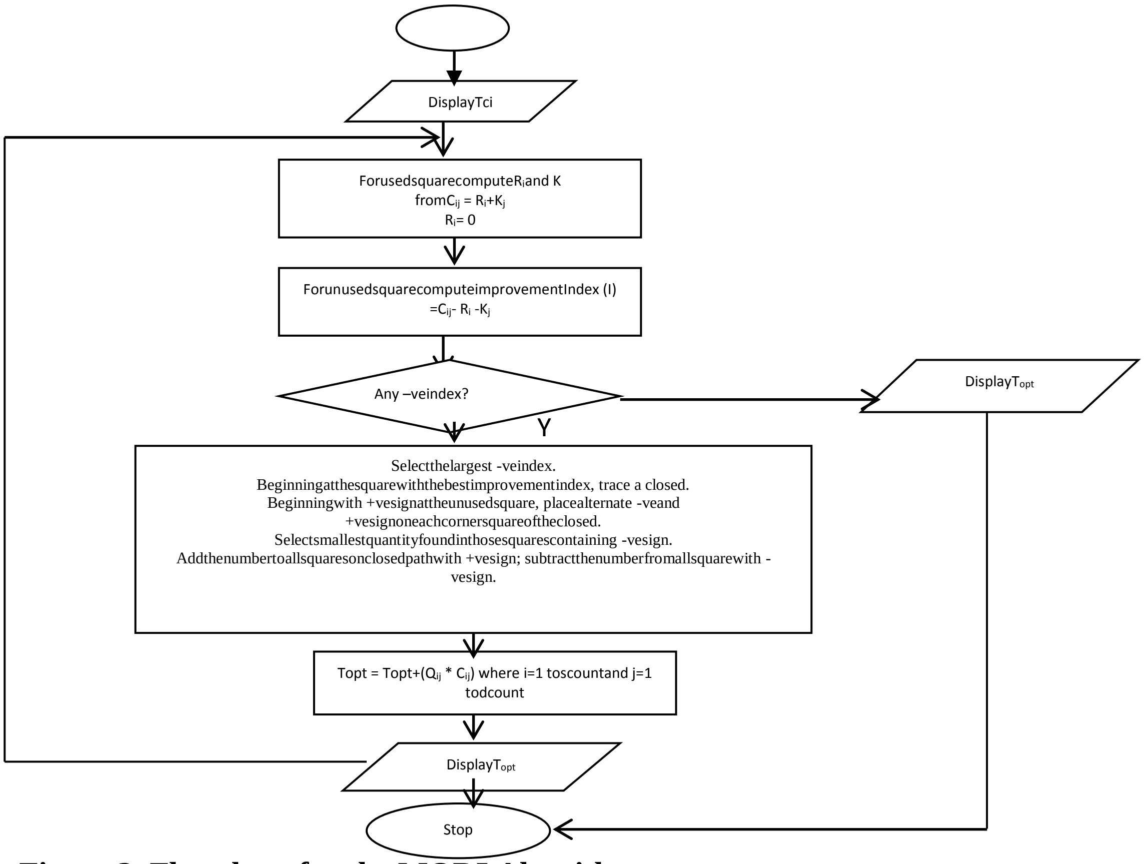


Figure 2. Flowchart for the MODI Algorithm

In the case study used for the implementation of the two programs in our case study, the supply capacity are 100 crates from depot A, 300 at depot B, and 300 from depot C. The demand station A requested for 300 crates, station B requested for 200 crates and station C requested for 200 crates. The distribution cost is given in a tabular form as given in Table 1.

Table 1. Distribution costs

	Warehouse A (₦K)	Warehouse B(₦K)	Warehouse C(₦K)
Depot A	5	4	3
Depot B	8	4	3
Depot C	9	7	5

Northwest Corner Program

North West class was created with Java programming language with its main method and six other methods which are: *getdata* method (the method responsible for getting input from user in rows and columns), *booleanbottleneck* method (the method that check if the problem is bottle neck or not), *distribution* method(this is the main method that is responsible for proper distribution of good from source(s) to destination(s)), *getCost* method (this method that get the cost of each of the distribution), *intcostcalculation* method (this is the method responsible for calculating the optimum cost of the distribution), and the *display* method which is responsible for the output of the distribution in a tabular form.

MODI Program

Modi2 class was created with Java programming language with its main method and nine other methods which are: *getdata* method (the method responsible for getting input from user in rows and columns), *booleanbottleneck* method (the method that check if the problem is bottle neck or not), *distribution* method(this is the main method that is responsible for proper distribution of good from source(s) to destination(s)), *getCost* method (this method that get the cost of each of the distribution), *intcostcalculation* method (this is the method responsible for calculating the optimum cost of the distribution), *improvement* method (this method provides an improvement over the NWC program), *reshuffle* method (this method reshuffle the table), *intsmallestcheck* method (this method check for the smallest number in an array of numbers), and the *display* method which is responsible for the output of the distribution in a tabular form.

Result and Discussion

The summary results of the given distribution program are presented in Table 2.

The comparative analysis was carried out based on average time of execution, the lines of code, number of methods, solution type, complexity of algorithm and complexity of the program based on the number of iterations and method call.

Average Runtime

i. For the northwest corner method, the average runtime is gotten from 5 program executions which are in nanoseconds (ns)
Execution time of program 1 = 835995 ns
Execution time of program 2 = 839843ns
Execution time of program 3 = 838561ns
Execution time of program 4 = 928872ns
Execution time of program 5 = 813371ns
AVERAGE RUNTIME = 4256642/5 = 851328.4ns

ii. For modified distribution method the average runtime is gotten from 5 program executions which are in nanoseconds (ns);
Execution time of program 1 = 2.1371428E7ns
Execution time of program 2 = 2.1920632E7ns
Execution time of program 3 = 2.2017511E7ns
Execution time of program 4 = 2.1753818E7ns
Execution time of program 5 = 2.1637133E7ns
AVERAGE RUNTIME = 108700522/5 = 21740104.4ns

Table 2. Comparison between NWC and MODI.

Program Class	**Average run time**	**Line of Code**	**No of Methods**	**Solution Types**	**Complexity of Algorithm**	**Complexity of Program**
Northwest Class	851328.4ns	157	6	Initial BFS (4200)	Simple	Simple
Modi2 Class	221740104.4 ns	408	9	Optimal Solution (4000)	Complex	Complex

The two programs were executed five times to calculate the average time spent on the execution. It was discovered that the MODI program spend more time in the execution due to more methods implemented by the class but the MODI produced the best Optimal solution. The NWC class did not give the best optimal class as the initial basic solution was 4200 and this was not the best solution as the solution gotten from MODI which was 4000.

Conclusion

From the above result, it was concluded that the Modified distribution index program is more efficient and accurate than the Northwest program of solving distribution problem as it provides the optimal solution. Though MODI gives a better solution, the development of the program in Java programming language is more complex.

References

Adeosun, A. (2010). Compiled lecture note on CSE 515, LadokeAkintola University of Technology, Ogbomoso, Oyo State, Nigeria.

Anderson, D., Sweeney, D., & Williams, T. (1995). *Quantitative Methods for Business* (6th ed.). St. Paul, Minn: West Publishing Company. Retrieved from http://www.swlearning.com/economics/mcguigan/mcguigan9e/web_chapter_b.pdf

Babu, A., Helal, A., Hasan, M.S., & Das, U.K. (2012). Implied Cost Method (ICM): An Alternative Approach to Find the Feasible Solution of Transportation Problem. *Global Journal of Science Frontier Research: F Mathematics and Decision Sciences*, *14*(1), 2249-4624.

Bhavya, N.S. & Arvind, K.S. (2016). Solving Transportation Problem using Excel Solver for an Optimal Solution. *MIT International Journal of Mechanical Engineering*, *6*(1), 18-20.

Hasan, M.K. (2012). Direct Methods for Finding Optimal Solution of a Transportation Problem are not Always Reliable. *International Refereed Journal of Engineering and Science, 1*(2), 46-52.

Hillier, F. S. & Lieberman, G.J. (1995). *Introduction to Operations Research*. New York, NY: McGraw-Hill Publishing Company.

Hong, Y., Vaidya, J., & Lu, H. (2011). Efficient Distributed Linear Programming with Limited Disclosure. *Journals of Data and Applications Security and Privacy XXV, 3*(2), 170–185.

Joshi, R.V. (2013). Optimization Techniques for Transportation Problems of Three Variables. *IOSR Journal of Mathematics, 1*(9), 46-50.

Kavitha, M. & Vinoba, V. (2015). A Study on Transportation problem using stepping stone method and its Applications. *International Journal of Mathematics Trends and Technology, 22*(1), 52-54.

Lord, M.S., Bazardeh, S.M., Khoshnood, S., Mahmoodi, N., Rasht-Abadi, F.Q., & Mohammadi, M.O. (2013). Linear Programming & Optimizing the Resources. *Interdisciplinary Journal of Contemporary Research in Business, 4*(11), 701-705.

Lovely Professional University, Directorate of Distance Education (2012). Operations Research. Retrieved July 13, 2016, from http://iimsnepal.com/download/E%20BOOK%20MATERIALS/MBA%20EBOOK%20MATERIAL/MBA%204th%20Semester%20Ebook%20Materials/DMGT504_OPERATIONS_RESEARCH.pdf

Maurya, V.N., Misra, R.B., Jaggi, C.K., & Maurya, A.K. (2014). Progressive Review and Analytical Approach for Optimal Solution of Stochastic Transportation Problems (STP) Involving Multi-Choice Cost. *American Journal of Modeling and Optimization*, *2*(3), 77-83.

Patel, R.G. & Bhathwala, P.H. (2013). The Advance Method for the Optimum Solution of a Transportation Problem. *International Journal of Science and Research*, *6*(14), 703-705.

Reeb, J. & Leavengood, S. (2002). Transportation Problem: A Special Case for Linear Programming Problems. *Journal of Performance Excellence in the Wood Products Industry (Operation Research).*

Singh, S., Dubey, G.C., & Shrivastava, R. (2012). A Various Method to Solve the Optimality for the Transportation Problem. *International Journal of Mathematical Engineering and Science*, *1*(4), 21-28.