Implications of Incorporating History in the Teaching and Learning of Mathematics Concepts in the Classroom: A Literature Review

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Abstract. The study of mathematics has largely been characterized by difficulties in understanding by students and unimpressive performance, especially in external examinations. As a result of this, the need to employ innovations in the teaching and learning of mathematics concepts has been recognized and has resulted in a variety of studies. This review explored how history can be incorporated into the teaching and learning of mathematics as a way of enhancing the mathematics instructional process. Mathematics just like any other field is characterized by a rich history, with several trends that resulted in its current state and nature. This literature also explored ways and approaches with which history can be incorporated into mathematics teaching and learning, the implications, potential benefits, and challenges. This review revealed that incorporating history in the teaching and learning process in the classroom can potentially enhance the mathematics learning outcomes, but there exist many challenges that might influence the process negatively, which need to be addressed before one can reap the beneficial results. Recommendations were proffered among others that curriculum planners should include the history of mathematics concepts in the mathematics curricula at all levels of education.

Keywords: History, performance, mathematics, teaching, and learning

Introduction

Mathematics is a fundamental field of study for life and daily living. It influences our daily lives and decisions in a variety of ways, and without its knowledge, one would be hard-pressed to live effectively in today's contemporary society. The importance and roles of mathematics cannot be quantified, as it only continues to increase, especially with the advancements of science and technology as it is primarily linked to their growth and utilization (Posamentier & Smith, 2020). Despite its acknowledged importance, mathematics is widely known to be a center of students' fear and anxiety and is characterized by trends of poor performance, especially when compared with other subjects (Bailey, 2017).

The outcome of this reported underwhelming performance especially in external examinations has led to a wide variety of studies exploring the factors responsible for those difficulties and how they can be breached (Butakor & Dziwornu, 2018; Acharya, 2017). Thus the students' difficulties in learning mathematics have over time been attributed to several causes and factors. One of the most popularly attributed factors responsible for students' difficulties with the subject is teachers' related factors. These teacher factors include variables like lack of experienced teachers, teachers' non-utilization of instructional resources, and the teachers' style of carrying instruction, etc. (Owan *et al.*, 2019). The recognition of the large roles teachers play in determining the learning outcomes has resulted in the need to introduce and incorporate innovation into their teaching style and process.

History is a fundamental aspect of any field of study, as no field can be entirely separated from its history. Learning and understanding history is associated with many benefits including understanding previous trends in the field, developing a better understanding of the nature of the field and the world, better understanding of present trends

and predictions for the future, becoming a more rounded individual in the field, understand the identity behind concepts in the present, inspirational tool, learning from mistakes and successes, developing transferrable skills, etc. (Balean, 2022; Nord, 2020). Considering the roles of history, it isn't too far-fetched to assume that it can potentially enhance the classroom instructional process.

Thus, this study was designed to explore the incorporation of history into the teaching and learning of mathematics in the classrooms as an innovation, how it can be used, and the implications.

Importance of Mathematics

The importance of mathematics can never be overstated and it has been a fundamental part of traditional western education since classical times. Mathematics is fundamental to and plays important role in our everyday lives (Bailey, 2017). It is described as a universal field of study covering a broad scope and as such is characterized by a wide variety of definitions, as no one definition has been able to capture the true essence of the mathematics field. Mathematics is often described as the scientific field of study that deals with the study of quantitative data, and primarily linked with abstract structures, and covers a broad scope including the number and their operations, relationships, order, structures, and various uncertainties (Ziegler & Loos, 2017). It emphasizes variables like computations, calculations, and logical reasoning, all geared towards the solving of problems.

The endless applications of mathematics to everyday life and living is one major variable responsible for its acknowledged importance and indispensability, which remains uncontended to date. It plays a significant role in our everyday life, that ranges from the obvious uses to the subtle (Emma, 2018; Sinay & Nahornicef, 2016). Mathematics influences and is responsible for simple everyday activities like tracking the time, money management, using a recipe, balancing a checkbook, etc. With the growth of science and technology in the world, the need for mathematical skills and knowledge has had significant growth, and individuals are being presented every day with scenarios that require the use of acquired mathematical knowledge or skills. Pua and Macutay (2020) affirm this stating that nowadays one would be hard-pressed to live in the modern contemporary society without an understanding of mathematics, as mathematics has become intertwined with the main elements and issues in our modern society, and as such is deeply rooted with the fundamentals of our modern society. Among others, it is a fundamental unit in national and personal security, people welfare, management of resources, economics, etc. Jayanthi (2019) states that a look through history shows that prioritizing mathematical knowledge and skills is a major contributor to fast-paced progress for any nation or society. Though this is sometimes silent or hidden, mathematics has helped shaped our world in a variety of ways and is also noted for its importance in revealing hidden patterns that help in understanding the world and society around us.

As a universal field of study, mathematics also exerts an amount of influence over a wide range of fields and domains, especially other science fields. Mathematical knowledge is fundamental to fields like engineering, finance, medicine, etc. The study of mathematics is considered important not just in its own right, but also for the associated benefits of knowledge which is usually needed and carried over to other knowledge domains. The Scientific World [TSW] (2018), states that the influence mathematics exhibits over other fields of study is limitless, as it virtually influences all forms of fields or knowledge domains no matter their nature or type, with them being dependent on it one way or another. Mathematics concepts and knowledge are also significant in all vocational fields and skills as individuals in those fields are faced daily with the quantitative process that is fundamental to carrying out their tasks and requires the knowledge of mathematics. These vocational fields

include fashion design, bricklaying, baking, carpentry, etc. (Dalley & Noyes, 2015). As the current contemporary society emphasizes scientific and technological advancements, mathematics and its applications have become a centerpiece for several successful careers and lives, and individuals with a concrete understanding of mathematics are in a better position to secure gainful employment and useful opportunities, with science being the vehicle of the present and the future (Gravemeijer *et al.*, 2017; Laura, 2021).

Mathematics is also fundamental to economic and scientific development. It has been severally described as the conceptual framework upon which modern science is based and is the language of science, technology, engineering, and research, and is indispensable in their creation, understanding, and continued advancement. It has exhibited a strong influence on science and technology and has been influenced by it in several ways. Thus, how developed a country is, is usually tied to its level of mathematical proficiency (Yadav, 2019; Wanjiru, 2015). It is also a fundamental tool in infrastructural development, thanks to its influence over all forms of engineering (Das, 2020).

All economic processes are also largely dependent on mathematics, and as such mathematics is fundamental to managing a nation's economy and commercial processes. It is also fundamental in managing any form of business through and used in a vast number of tasks including accounting, marketing, financial analysis, management of inventory, calculation of variables like hire rates and taxes, etc. (Patil, 2020; Deeb, 2019).

Mathematics is also important in the building up of the cognitive faculties of an individual. VaraidzaiMakondo and Makondo (2020) state that mathematics aids in the proper building up of human thought and logic. Mathematics has been linked to several psychological and intellectual benefits, this includes the development of problem-solving skills and insights, creativity, critical, analytical, fast thinking and reasoning, and even communication skills (Jayanthi, 2019). The contribution of mathematics to building up the mental and intellectual framework is primarily attributed to the active mental work required in solving mathematical problems.

Khan (2015) states that the scope of mathematics benefits goes beyond just visible everyday problems, with it also being fundamental in developing imaginations, intuitions, and reasoning that can be used in finding new ideas and solving puzzling problems. Also, the study of mathematics and its understanding is deemed to be personally satisfying and empowering for the individual with mathematicians often citing the essence of mathematics as lying in the beauty of numbers, figures and relations.

Students' Performance in Mathematics

Several studies have over time been carried out to assess the level of students' achievement in mathematics and the influencing factors affecting that achievement, especially at the secondary school level. A good performance at the secondary school level is of the utmost importance as it determines to a large extent the success or failure of the whole educational system, as it acts as the basis for career building and future academic studies with the tertiary education level being built upon it (Donald & Richard, 2016; Awual *et al.*, 2014). Secondary school is a great determinant and predictor of future academic performance and as such a good performance at the secondary school level is a matter of necessity, especially for core subjects like mathematics which determines whether the students can go for further studies at tertiary institutions of learning, no matter the field of study.

Ewetan and Ewetan (2015), state that for any nation that wants to achieve rapid scientific, economic and technological development, good performance of its citizens in mathematics is a matter of necessity. However, despite the recognition of the importance of mathematics and the position it occupies in building up the nation and in everyday life, findings of studies have always reported it as one of the most difficult subjects for students to

learn and is characterized by a trend of underperformance (VaraidzaiMakondo & Makondo, 2020).

Awofala (2017), states that results from both internal and external examinations had always shown that Nigerian secondary school students had always had a trend of poor performance in the subject with the majority not demonstrating a deep understanding of mathematical concepts and capacity to do mathematics. Ewetan and Ewetan (2015) assessed the West African Examination Council (WAEC) performance of students in some selected secondary schools in mathematics and reported it to largely be on the poor side.

Idowu (2016) also affirms that despite the recognized indispensability of mathematics, which led to it being established as a core and compulsory subject, and the attempts made to enhance the mathematics learning process, there had been little to no improvements in the students' performance. Owan *et al.* (2019) state that the poor performance in mathematics has led to parents and other stakeholders in the educational institutions, questioning the effectiveness of the secondary school classroom instruction in Nigeria.

The students' unimpressive performance in mathematics has over time been attributed to a wide variety of reasons and these include the learners' attitude towards the subject, lack of experienced mathematics teachers, lack of proper instructional facilities and equipment, teaching styles/instructional strategies, abstract nature of mathematics concepts, etc. (Acharya, 2017; Butakor & Dziwornu, 2018).

With the recognition of the role the teachers and the teaching style play in determining the students' learning outcomes in mathematics, it has become the need for the introduction or incorporation of other or new innovative strategies into the teaching of mathematics have been continually recommended. This study thus explores the use of history, as an innovative tool, that can be incorporated into mathematics teaching.

Incorporating History into the Teaching and Learning of Mathematics: An Overview

Human existence cannot do without history. This is also applicable to all fields of study including mathematics. Thus, the present mathematics is determined by the past mathematics. Vittori (2018) states that the history of mathematics is an important component in the learning of mathematics, and the relevance of history in mathematics teaching and learning has continually been reaffirmed by a wide variety of researchers all over the world. Azman and Maat (2021) reaffirm this view stating that incorporating the history of mathematics into mathematics teaching and learning can enhance the efficacy of the learning process and the learning outcomes.

The incorporation of history into the teaching and learning of mathematics can exist in two forms; history as a tool (where the historical contents are incorporated into teaching as a strategy to enhance the mathematics teaching and learning process), or history as a goal (concerned with giving students the actual knowledge of mathematical histories, it's development, interaction with other fields or domains, etc. (Kapofu & Kapofu, 2020; Astin *et al.*, 2016).

There is a major absence of recent relevant studies exploring the use of history in the mathematics teaching and learning process as it is not a common recommendation or strategy. Vittori (2018) states that since the beginning of the 21st century, the international community of people interested in incorporating history into the teaching of mathematics has been calling for more empirical studies that could bring into light just how effective it is, and the role historical elements can play in mathematics teaching but little work has been done about it.

Considering the nature of history and its linkages to all academic fields, many potential benefits have been associated with its incorporation into the teaching and learning process of

school subjects like mathematics. Azman and Maat (2021), state that the proper incorporation of history into mathematics instruction/teaching sessions aids in the development of effective and cognitive skills as well as empowering students to understand the true nature of mathematics. According to their study, the findings of most (of the few available) studies have largely reported positive effects on the students, teachers, and the entire mathematics teaching and learning process, when history was used in carrying out instruction or teaching.

Burns (2010), in his study, implied that the incorporation of history into the teaching and learning of mathematics is largely advantageous. The study cited some benefits to be derived through the incorporation of history into the teaching and learning of mathematics. Amongst others they include enabling students to learn the evolution of mathematics as a subject and its concepts, allowing the students to make connections, sharpening problem-solving skills by introducing them to diverse approaches used throughout history and a variety of algorithms and techniques, highlighting the interactions between mathematics and society and the effect of those interactions. The study further identifies some other benefits of incorporating history into the teaching and learning of mathematics through the view of pre-service teachers. One of the listed benefits at the end of the study was an improvement in an individual's knowledge as participants in the study indicate that they enjoyed the lessons and assignments, and the mathematics history incorporation process encourage the participants to do some research on their own on mathematicians in history as well as the "when, whys and hows" of some mathematical symbols, theorems and patterns they must have encountered and used. Some of the participants in the study even noted that the assignments and learning process forced them to learn more about current trends and people in mathematics rather than only figures from ancient times. Other participants in the study state that the incorporation of history into the teaching of mathematics led to them getting a better understanding of how different mathematical concepts relates to each other and learning mathematical logic. Other cited benefits (by the pre-service teachers) of incorporating history into the teaching of mathematics include; an increase in the students' motivation and a greater appreciation for mathematics, expansion of teaching styles (as some students will learn better with the use of stories and fragments from history), creation of interdisciplinary activity opportunities as the students make connections, etc.

Implications of Incorporating History in the Teaching and Learning of Mathematics Concepts

A major implication of the incorporation of history into the teaching and learning of mathematics is that it can be utilized in such a way as to ensure the maintenance of the true essence and meaning of the subject. This implies that separating subjects like mathematics from the historical elements of time and human society strips away a layer of meaning of the subject, along with its true essence. Teaching mathematics without the relevant historical contexts cannot fully imbibe in the learner the true understanding of what mathematics is and stands for, what makes mathematics true and why, what makes it beautiful, what contributed to its existence, and its true importance in shaping our society. Thus, the incorporation of history into the teaching and learning of mathematics adds those needed layers of meaning and knowledge.

With the unchallenged understanding that history is the most fundamental aspect in the building up of any subject to its current state, there is an implication that the incorporation of history into the teaching and learning of subjects like mathematics can help break down the overall complexity and novelty of the subject. Using history in the teaching of mathematics helps present the subject as one characterized by continuous improvement and several variables that contributed to its current state and status, and not just a complex collection of truths and quantifiable data and problems, that remain unchanged and unadaptable to trends

in time and society (Dejic & MIhajilovic, 2015). This as noted earlier, implies that history is needed to give mathematics students a true understanding of the nature and content of the subject. Basibiyuk and Sahin (2019) state that incorporating history into the mathematics teaching and learning process equips the students with fundamental knowledge of the needed historical context of mathematics, including the origins of the concept being addressed and in creating linkages between mathematics concepts to everyday life, including developing meaningful relationships between abstract mathematical ideas and practical applications in a real-world context.

Understanding the historical context of mathematics also implies the development of more critical points of view of not just the students but also the teacher on the mathematical contents especially its relation to the culture and history of the society. Mathematics on its own has been noted to develop the mental faculties, including in areas that concern critical thinking (Tan-Sisman & Genckaya, 2021), and as Li and He (2020) assert, the use of history in teaching mathematics makes it more effective in helping the students in thinking and constructing ideas by themselves while also exploring the true beauty of the subject. This implies incorporation of historical elements into the actual teaching of mathematics develops further the mental faculties that have to do with critical and creative thinking, while also providing the individual with more points of view of the subject itself.

The use of history in teaching has been linked to increased efficacy in learning outcomes (Vittori, 2018). This implies that the incorporation of history into mathematics creates room for better and enhanced approaches to teaching, and understanding mathematical concepts and their relation to present world variables, with an emphasis on making the actual learning more engaging and interesting. A possible scenario of this is the students and teachings comparing current trends or teaching strategies to those set originally in the past. This is an interesting way to understand several important variables like the contents of the concept being learned, or the effectiveness of current mathematical processes, including its impact on things like the economy of modern methods, especially when compared to ancient times, and understanding the historical evolution of a mathematics concept. Barbin *et al.* (2018) and Baki and Gursoy (2018), assert that the incorporation of history into the teaching and learning process makes for a better approach to teaching and it is a potential tool for enhancing the students' classroom engagements, interest in the subject matter, arousing the students' curiosity and in helping them develop positive attitudes towards the learning of mathematics. In line with this Butuner and Baki (2020) state that inculcating history into the mathematics, the teaching process is linked to many benefits, among which include an improvement in the learners' motivation and positive believes and attitudes towards mathematics, as well as readiness to learn mathematics.

Galante (2014) states that the use of history will also lead to innovations in teaching strategies. This implies that the teacher through their exposure to the historical content of mathematics as well as its teaching through time, can adopt innovative ways of teaching the subject and that teaching strategies and activities can be designed to be centered around and accommodate the incorporation of history into the mathematics teaching process.

Another implication of the introduction of history into mathematics teaching is that the teacher can use it as a platform to create an understanding of the backbone to support the teaching of the current mathematics concepts. History can be used to justify why certain mathematical concepts are still present in the current curriculum.

How History can be Incorporated into the Teaching and Learning of Mathematics Concepts

Though literature concerning the actual incorporation of history into mathematics teaching and learning is largely limited, there exists a number of them exploring techniques

and approaches towards the incorporation, along with some practical examples. Fried *et al.* (2016) cited the hermeneutic approach as an example of a good pedagogical approach to integrating history into the teaching of mathematics concepts. The principles of this approach can be summarized in some steps. The students first study the historical background of a concept or topic after acquiring a good understanding of the said mathematical topic in its contemporary/modern form. The students then gather information and contexts concerning the biography of the author, while keeping the historical peculiarity of the source far away. The teacher then asks the students with providing reasoned arguments while rejecting general interpretations. Finally, the historical understanding of the concept or topic is contrasted with the modern view, encouraging the process of reflection.

Azman and Maat (2021) list three approaches for the integration of history into the mathematics classroom situation and include; learning mathematics history and supporting mathematics learning through providing direct information on history, using historical-inspired teaching strategies, and developing deepened awareness of the social and cultural contexts of mathematics and its linkage to practical life. Tools and ideas that can be used in the history of mathematics teaching incorporation process include; the usage of historical events in teaching, conducting mathematics history-based projects, incorporating of history elements in questions, discussing problems from or related to the historical contents of mathematics, discussion of historical issues and arguments related to mathematics, etc.

Other ways to incorporate history in mathematics teaching include among others using the historical content of Goldhach's conjecture, Gause number theory, and discussion of the declaration of independence to teach geometry. Also, using the historical content of numerals in ancient number systems to teach basic mathematics, using the historical context of the Babylonian guess-and-check method for solving equations, using Al-khusarizm's completing the square, solving right triangle problems from historical documents to teach Algebra. (Clark, 2015). Despite many challenges encountered, the findings of the study reported generally favorable results with the teachers in the study feeling more empowered after using history to teach mathematics for only a semester. The study further categorized how history can be used in a mathematics classroom setting and this includes using history as an anecdote, history as a biography, and, history as an interesting problem.

Furner and Brewer (2016), explored ways history can be used to enhance the teaching and learning of mathematics using several mathematics concepts as an example or case study including topics like numbers and numerals, measurement, data analysis, probability, geometry, etc. When dealing with numbers, students could be introduced to their origin. The Hindu-Arabic numerals are the one that is used internationally today and originated from the Indian numerals. The current number system came about through the transmission of numerals across several nations and empires, with the numerals continually being modified in shape as it is passed along before eventually settling on their final current shape. The use of Arabic numerals spread through European books, colonialism, etc. (Danna, 2019). To enhance the learning of numbers the students could be introduced to the interesting backgrounds of numbers such as this, as these stories are bound to induce much more engagement and interest from the students than if they are just taught the mathematics of numbers independent of their historical background. Students may also enjoy exploring the different number systems from different cultures and periods. Another historical item that can be incorporated into the teaching of numbers is the use of Napier Bones/Rods which originated in the 1700s and were created by John Napier from Scotland (Sparrow, 2015; Dave, 2020). Making use of such archaic items might result in enhancing the students' curiosity and interest and these rods can also help them to better understand mathematics, place value, and lattice multiplication.

The history of the metric system which originated in France around the 1700s can be incorporated into the teaching of measurement in the classroom. The origins of measurement are also characterized by some stories that the students might find interesting and enhance the teaching and learning process. Before the units of measurements were created, primitive societies and individuals made use of body parts like the foot, index finger, the tip of the chin outstretched fingers, etc. as their measurement tools. The "foot" as a unit of measurement also originated from the ancient Greeks and its history is linked with a myth that it was based on the actual measurement of Hercules' (Demigod son of the god of lightning, Zeus) foot (Zupko, 2018). Romans created the mile, the French created the metric system in 1790, the English Parliament legalized the yard in 1760, etc. Teachers can use a wide number of interesting facts and history and incorporate them to make class learning even more interesting and engaging for the students. The students their part are much more likely to retain and recall learned knowledge and all of the mathematics if the learning process is accompanied by such interest-inducing historical content.

Data analysis and probability also have quite an interesting history, which can be used to pique the students' attention and interest in the topic. It has its roots in Italy, where the individuals were trying to improve their chances at card and dice games, and has its origins linked to well-to-do gamblers, prominent mathematicians who calculated the odds for a variety of games (Porter, 2020; Camilleri, 2018). Taking advantage of this knowledge, teachers can also enhance probability learning through the use of games like cards and dice games.

The origin of Geometry is primarily linked to Greece around 2000 B.C. and teachers can also take advantage of its interesting past to enhance its teaching in the classroom. Furner and Brewer (2016) state that geometry was considered among Greeks and the Mediterranean regions to be quite the important subject that signs would be hung in university entrances saying "Let no man ignorant of geometry enter here." This is just a fragment of its history, as geometry has quite a rich historical past. When handling geometry classes, the teachers can introduce these historical fragments and incorporate them into the lesson plan.

Factors Mitigating Against the Incorporation of History into the Teaching and Learning of Mathematics

The proper incorporation of history into the mathematics teaching and learning process is infused with many problems or obstacles. One of the major obstacles to its implementation has to do with time. It takes a deal of time to study and get the needed understanding of those relevant histories and adapt the content for use with the mathematics students. The time constraint problems also include the limited time allocated for lessons in the classrooms, as the teachers may not have enough time to start introducing history in their lectures (Baki & Gurosey, 2018; Bucholtz & Schorcht, 2019).

Another major issue associated with the use of history in the teaching of mathematics is the teachers' knowledge of the historical context of the subject. The knowledge needed by mathematics teachers for the seamless integration of history into mathematics teaching also involves being able to select appropriate and reliable resources. It is quite unfortunate then, that the common trend is that though the teacher might have a good understanding of mathematics concepts and working, they know little to no content related to the background or history of mathematics, as these are rarely taught even in teacher training. Let alone being trained or capable on how to adapt these wild histories to the classroom teaching and learning process (Kusumawati *et al*., 2020; Wang *et al*., 2018).

Reports from studies showed that though most of the teachers had positive perceptions of the use of history in the teaching of mathematics and its benefits, their self-efficacy in carrying out the task is quite on the low side. Even though the teachers had positive views

about the potential benefits to be derived from the process, most did not consider applying it. The reluctance to apply history to mathematics classrooms can be linked to the poor knowledge of the historical aspects of mathematics by the teachers (Butuner, 2018; Basibiyak & Sahin, 2019). Bucholtz and Schorcht (2019) state that a sizeable number of teachers deemed the process involved and needed for the incorporation of history in teaching mathematics as being too complex.

Another factor contributing to difficulties in incorporating history into the teaching of mathematics is the lack of relevant materials and resources to guide the process (Furner & Brewer, 2016). There might be a major lack of studies exploring the use of history in teaching especially concerning how to implement it in the classroom. This raises a situation where even if the teacher desires to adopt the strategy, they would be hard-pressed to carry it out effectively, as they might be doing so blindly and experimentally, without a prior understanding of what works and what doesn't. Though teachers might express interest in the incorporation of history in teaching mathematics, they are faced with difficulties as they have no idea where or how to start. This issue comes about, due to the lack of relevant material resources to guide the teachers.

Also linked with the lack of available materials, is its exclusion from the school mathematics syllabus (Butuner, 2018), which in turn contributes to its lack of implementation in the classroom situation. Also, the incorporation of history into mathematics teaching requires a great deal of adequate planning, if it is to succeed, and if not planned properly is easily prone to failure.

Conclusion

History is a major constituent of any field of study, as the trends in history are responsible for the current state and nature of the subject. This study explored the implications of incorporating history in the teaching and learning of mathematics concepts in the classrooms. Several ways and approaches can be employed in incorporating history into the teaching and learning of mathematics, and the potential benefits in the classroom situation. History can be utilized as a tool to enhance the teaching and learning of mathematics but the process is identified with several challenges that the teacher will have to overcome if the history is to be effectively incorporated to enhance learning outcomes in mathematics.

Recommendations

Based on the study, the following recommendations are made;

1. Curriculum planners should incorporate the history of mathematics concepts in mathematics curricula at all levels of education.
2. Material resources should be provided for teachers to teach and learning of historical facts in mathematics.
3. Provision of the teacher's guide on the approaches to teaching the history of mathematics should be made available in the teaching and learning of mathematics at all levels of education.
4. Mathematics educators should conduct seminars and workshops to enhance teachers' knowledge of the ways to incorporate the history of mathematics in the teaching and learning process.

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