About the Calculus of Cantilever Dental Bridge with a Single Pole in Medial Extension by Transfer-Matrix Method

Mihaela Suciu
Department of Mechanical Engineering,
Technical University of Cluj-Napoca, Cluj-Napoca, Romania

Abstract: This paper presents an original and interesting approach for calculus of cantilever dental bridge with a single pole in medial extension. Orthodontic applications of mathematical algorithm for calculus in dentistry are in continuous development. We present a calculus of efforts (bending moment and cutting force) and deformations (arrow and rotation) in a cantilever dental bridge with a single pole in medial extension, that is a different, but an original approach for orthodontic problems about cantilever dental bridge studies. We treat this problem by assimilating the cantilever dental bridge with a cantilever beam, embedded at the left end and free at the right end. It is very important for bio-engineering application because the calculus of beams is well known by specialists in field. The idea of applying the approach by Transfer-Matrix Method (TMM) is original and very easy and interesting to dental bridge calculus as a beam. A medial extension is an extension of anterior part of mouth, in contrast to a distal extension consisting in an extension to posterior part of oral cavity. At first, we present a beam calculus using the facilities given by the functions and operators of Dirac and Heaviside. After, we applied the TMM for calculus of dental bridge with medial extension as a beam. TMM is very easy to program, and the results can be used rapidly in situ by dentists in special conditions. We hope for the future to present validation of results by Finite Elements Method and by experimental research too.

Key words: Dental Bridge, Medial Extension, Charge Density, State Vector, Transfer-Matrix Method (TMM)

Introduction

Orthodontic applications of mathematical algorithm for calculus in dentistry are in continuous development. We present an original and interesting approach for calculus of cantilever dental bridge with a single pole in medial extension, a calculus of efforts (bending moment and cutting force) and deformations (arrow and rotation) in a cantilever dental bridge with a single pole in medial extension. It is a different, but an original approach for orthodontic problems about cantilever dental bridge studies. We treat this problem by assimilating the cantilever dental bridge with a cantilever beam, embedded at the left end and free at the right end. This similarity is very important for bio-engineering application because the calculus of beams is well known by specialists in field. A medial extension is an extension of anterior part of mouth, in contrast to a distal extension consisting in an extension to posterior part of oral cavity. At first, we present a beam calculus using the facilities given by the functions and operators of Dirac and Heaviside.

Literature Review

Due to the importance of the masticatory apparatus, there are many studies in the field, especially from the point of view of dentist specialists. Next, we will present some of these.

A study for ten-year survival of bridges placed in the General Dental Services in England and Wales is presented in (Burke & Lucarotti, 2012). In (Dikova & Vasilev, 2019), we have simulation analysis and tests about bending fracture of Co-Cr dental bridges, produced by additive technologies and in (Dikova, 2018) we have experimental investigations about *bending* fracture of Co-Cr dental bridges, produced by additive technologies: experimental

investigation. An integrated construction and simulation of tool paths for milling dental crowns and bridges is presented in (Gaspar & Weichert, 2013). The basics of the TMM calculation are given in (Gery & Calgaro, 1987). The importance of studies for dental bridges results from (Pjetursson et al., 2007) which make a comparison of survival and complication rates of tooth-supported fixed dental prostheses (FDPs) and implant-supported FDPs and single crowns. About design implications for dental bridge geometry due of fatigue of zirconium is presented in (Quinna et al., 2010). Some comparative retrospective case series about implant-supported 2-unit cantilevers compared with single crowns on adjacent implants is presented in (Roccuzzo et al., 2020). A review study about assessment of various factors for feasibility of fixed cantilever bridge is given in (Sharma et al., 2012). It can be seen a comparison of using different bridge prosthetic designs for partial defect restoration through mathematical modeling in (Styranivska, Kliuchkovska & Mykyyevych, 2017). Study about fatigue of zirconium under cyclic loading in water and its implications for the design of dental bridges is given in (Studart et al., 2007). Fundamental bases of the calculation in resistance of materials are given in (Suciu & Tripa, 2021). Another approach by digital image correlation and finite element analysis of bone strain generated for implant-retained cantilever fixed prosthesis it can be seen in (Tribst et al., 2020). We have a study about the analytical calculus of dental bridge with distal extension and single pole tooth assimilated as a beam by TMM in (Tripa et al., 2020). Some contributions on the analytical calculus of simple dental bridge assimilated with a beam embedded at both ends by TMM are presented in (Tripa et al., 2019). Similarity for dental bridges as frames calculated by TMM is given in (Tripa et al., 2019). An exhaustive presentation of formulas for stress and strain can be found in (Warren, 1989).

Materials and Methods

This work presents a study by TMM based on the facilities offered by calculus with Dirac's and Heaviside's functions and operators for a cantilever dental bridge with medial extension assimilated as a beam, embedded at left end and free at right end with vertical concentrated loads. The algorithm can be very easy to program.

The idea of applying the approach by Transfer-Matrix Method (TMM) is original and very easy and interesting to dental bridge calculus as a beam.

Cantilever Dental Bridge with a Single Pole

For practical applications in bio-engineering problems, dental bridge assimilated with a beam is an approach very important for orthodontic fields. The dental bridge is necessary when it is affected the integrity of chewing apparatus, when one or more teeth are missing, for partial edentulous, which affects its three functions: mastication, phonetics, and aesthetics. The dental bridges are fixed prostheses. A dental bridge is formed by poles, aggregation elements and the body of dental bridge. A medial extension is an extension of anterior part of mouth, in contrast to a distal extension consisting in an extension to posterior part of oral cavity. Poles can be natural existing teeth, capable to supporting aggregation elements, or dental implants. We study a cantilever dental bridge with a single pole in medial extension, as shown in Figure 1. Aggregation element and body of dental bridge are made together and can thus be similar to a beam. For our cantilever dental bridge, as in Figure 1, we have a single pole, the aggregation element is stiffened on the pole tooth, allowing assimilation of the stiffener with embedded support, at the left end of dental bridge. Dental bridge must be constructed so that at maximum loads, the bridge body will not touch the gum. We consider a dental bridge consisting of one aggregation element and a single bridge body for a single missing tooth (Figure 1). The most unfavorable requests are given by two equal concentrated vertical loads P, which act as follows: one in embedded section, the left end of cantilever dental bridge, on the aggregation element and the other, in middle of section of dental bridge body (Figure 1).

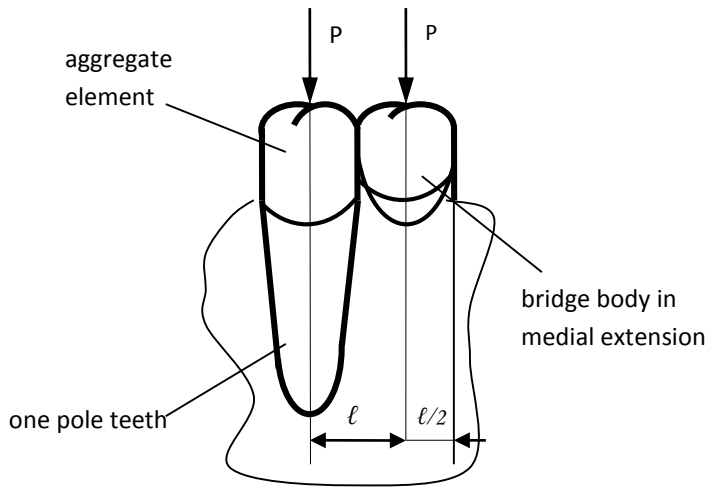

Figure 1. Cantilever dental bridge with one pole in medial extension.

We considered the cantilever dental bridge with medial extension and with a single pole as a beam, as shown in Figure 2.

Cantilever Beam Studied with Dirac's and Heaviside's Functions and Operators

In Figure 2, we have a cantilever beam, with which the cantilever dental bridge was assimilated. The beam is embedded at left end and free at the right end with two vertical equal loads. That is the model for a cantilever dental bridge with a single pole in medial extension.

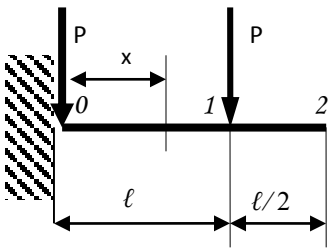

Figure 2. Cantilever dental bridge with medial extension as a beam embedded at a left end and free at the right end with two concentrated vertical loads at the middle of teeth.

The charge density function corresponding to the two concentrated vertical loads is (1):

$$q(x) = -P\delta(x) - P\delta(x-l) \tag{1}$$

The mathematical algorithm used is given by the functions and operators of Dirac and Heaviside. We consider the arrow *y(x)*, the first derivative of the arrow being the rotation *ω(x)*, (2):

$$\frac{dy}{dx} = \frac{1}{EI}\omega(x) \tag{2}$$

Equation of the deformed average fiber is (3), which gives the bending moment *M(x)*, (Suciu & Tripa, 2021), that is:

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\frac{d\omega}{dx} = -\frac{M(x)}{EI} \tag{3}$$

Deriving (3) once more, we obtain (4), the cutting force *T(x)*:

$$\frac{d^3y}{dx^3}=\frac{1}{EI}\frac{d^2\omega}{dx^2}=-\frac{1}{EI}\frac{dM(x)}{dx}=-\frac{1}{EI}T(x) \tag{4}$$

Derived once again (4), we obtain (5), the uniformly distributed load *q(x):*

$$\frac{d^4y}{dx^4}=\frac{1}{EI}\frac{d^3\omega}{dx^3}=-\frac{1}{EI}\frac{d^2M(x)}{dx^2}=-\frac{1}{EI}\frac{dT(x)}{dx}=\frac{1}{EI}q(x) \tag{5}$$

Now, (1) is replaced in (5) and we obtain (6):

$$\frac{d^4y}{dx^4}=-\frac{P}{EI}\delta(x)-\frac{P}{EI}\delta(x-l) \tag{6}$$

Expression (6) is integrated successively four times, considering the operating rules of Dirac and Heaviside, which gives (7) for *T(x):*

$$\frac{d^3y}{dx^3}=-\frac{P}{EI}Y(x)-\frac{P}{EI}Y(x-l)+C_1 \tag{7}$$

where:

$$Y(x)$$

and:

$$Y(x-l)$$

are the Heaviside's functions.

Integrating once more (7), we get the moment *M(x)*, (8):

$$\frac{d^2y}{dx^2}=-\frac{P}{EI}xY(x)-\frac{P}{EI}(x-l)Y(x-l)+C_1x+C_2 \tag{8}$$

It is integrated once more (8) and it is obtained (9), the rotation *ω(x)*:

$$\frac{dy}{dx}=-\frac{P}{2EI}x^2Y(x)-\frac{P}{2EI}(x-l)^2Y(x-l)+C_1\frac{x^2}{2}+C_2x+C_3 \tag{9}$$

It is integrated once more (9) and it is obtained (10), the arrow *y(x)*

$$y(x)=-\frac{P}{6EI}x^3Y(x)-\frac{P}{6EI}(x-l)^3Y(x-l)+C_1\frac{x^3}{6}+C_2\frac{x^2}{2}+C_3x+C_4 \tag{10}$$

where C_1, C_2, C_3, C_4 are integration constants and which are determined putting the following conditions at the ends of the beam, as shown in Figure 2:

· in the point *0*, for *x = 0*, we have:

$$y(0)=y_0=0 \tag{11}$$

· in the point *0*, for *x = 0*:

$$\frac{dy}{dx}(0)=\omega(0)=\omega_0=0 \tag{12}$$

· in the point *2*, for:

$$x=\frac{3l}{2}$$

we have:

$$\frac{d^3y}{dx^3}\left(3\frac{l}{2}\right)=T\left(3\frac{l}{2}\right)=0 \tag{13}$$

· in the point *2*, for:

$$x=\frac{3l}{2}$$

we have:

$$\frac{d^2y}{dx^2}\left(3\frac{l}{2}\right)=M\left(3\frac{l}{2}\right)=0 \tag{14}$$

Integration constants C_1, C_2, C_3 and C_4 will be calculated with Dirac's and Heaviside's functions and operators (Gery & Calgaro, 1987), and we obtain:

· from (11):

$$C_4=0 \tag{15}$$

· from (12):

$$C_3=0 \tag{16}$$

· from (13):

$$C_1=-\frac{2P}{EI} \tag{17}$$

· from (14):

$$C_2=\frac{5Pl}{EI} \tag{18}$$

The calculated integration constants C_1, C_2, C_3 and C_4 are replaced in the expressions of $y(x)$, $\omega(x)$, $M(x)$ and $T(x)$ and we get the relations:

- (19) for the arrow $y(x)$:

$$y(x)=-\frac{P}{6EI}x^3Y(x)-\frac{P}{6EI}(x-l)^3Y(x-l)-\frac{2P}{EI}x^3+\frac{5Pl}{EI}x^2 \tag{19}$$

- (20) for the rotation $\omega(x)$:

$$\omega(x)=-\frac{P}{2EI}x^2Y(x)-\frac{P}{2EI}(x-l)^2Y(x-l)-\frac{2P}{EI}x^2+\frac{5Pl}{EI}x \tag{20}$$

- (21) for the moment $M(x)$:

$$M(x)=-\frac{P}{EI}xY(x)-\frac{P}{EI}(x-l)Y(x-l)-\frac{2P}{EI}x+\frac{5Pl}{EI} \tag{21}$$

- (22) for the cutting force $T(x)$:

$$T(x)=-\frac{P}{EI}Y(x)-\frac{P}{EI}Y(x-l)-\frac{2P}{EI} \tag{22}$$

At this moment, traversing the cantilever beam with the variable x, the values of the two deformations: the arrow $y(x)$ and the rotation $\omega(x)$ and the two efforts: the bending moment $M(x)$ and the shear force $T(x)$, can be calculated in any section of the beam.

With the results obtained for the efforts, the normal and tangential stresses can then be determined in the beam and, by analogy, in the cantilever dental bridge with a single pole in medial extension.

Cantilever Beam Studied with a Single Pole in Medial Extension as a as a Beam by Transfer-Matrix Method

As shown in Figure 2, the cantilever beam can be calculated by an analytical approach with TMM, based on theory of functions and operators of Dirac and Heaviside (Gery & Calgaro, 1987). We will use as a working hypothesis that we will consider having only two efforts: the bending moment M and the cutting force T. We consider a hypothesis too: the exterior vertical loads P acting in the points *0* and *1*, at the middle of teeth, they have the charge density given by expression (1).

For the cantilever dental bridge (as in Figure 1.) and likewise for the cantilever beam as in Figure 2, the following state vectors can be defined:

- for a certain section x, the state vector with four elements is defined as follows (23):

$$\{V(x)\} = \{V\}_x = \{y(x), \omega(x), M(x), T(x)\}^{-1} \tag{23}$$

- for the left end, the embedded end, that is noted with *0* – the origin, we have the state vector from the origin for *x=0*, we have (24):

$$\{V(0)\} = \{V\}_0 = \{y_0, \omega_0, M_0, T_0\}^{-1} \tag{24}$$

- for the right end, the free end, that is noted with *2*, the last section, for:

$$x = \frac{3l}{2}$$

we have the state vector for last section, we have (25):

$$\left\{V\left(\frac{3l}{2}\right)\right\} = \{V\}_{\frac{3l}{2}} = \left\{y\left(\frac{3l}{2}\right), \omega\left(\frac{3l}{2}\right), M\left(\frac{3l}{2}\right), T\left(\frac{3l}{2}\right)\right\}^{-1} \tag{25}$$

The following notations will be used:

- $\{V\}_x$ - the state vector for the section *x*;
- $[M]_x$ - the Transfer-Matrix between the section *0* and the section *x*;
- $\{V_e\}_x$ - the state vector for the free term at the section *x*, which depends on external loads;
- *E* - the Young modulus or the modulus of longitudinal elasticity;
- *A* - area for transversal section of beam;
- *I* - moment of inertia.

Matrix relation between the state vector of section *x* and the state vector of origin section *0*, $\{V\}_x$ is (26):

$$\{V\}_x = [M]_x \{V\}_0 + \{Ve\}_x \tag{26}$$

Mathematical algorithm is presented in (Gery & Calgaro, 1987). We can write with relations (19), (20), (21) and (22), expressions (27):

$$\begin{cases} y(x) = y_0 + \omega_0 x + \frac{M_0}{2EI}x^2 - \frac{T_0}{6EI}x^3 - \frac{P}{6EI}x^3 Y(x) - \frac{P}{6EI}(x-l)^3 Y(x-l) - \frac{2P}{EI}x^3 + \frac{5Pl}{EI}x^2 \\ \omega(x) = \omega_0 + \frac{M_0}{EI}x - \frac{T_0}{2EI}x^2 - \frac{P}{2EI}x^2 Y(x) - \frac{P}{2EI}(x-l)^2 Y(x-l) - \frac{2P}{EI}x^2 + \frac{5Pl}{EI}x \\ M(x) = M_0 - T_0 x - \frac{P}{EI}xY(x) - \frac{P}{EI}(x-l)Y(x-l) - \frac{2P}{EI}x \\ T(x) = T_0 + \frac{P}{EI}Y(x) + \frac{P}{EI}Y(x-l) - \frac{2P}{EI} \end{cases} \tag{27}$$

(27) can be transcribed matrixally as (28):

$$\begin{Bmatrix} y(x) \\ \omega(x) \\ M(x) \\ T(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & \frac{x^2}{2EI} & -\frac{x^3}{6EI} \\ 0 & 1 & \frac{x}{EI} & -\frac{x^2}{2EI} \\ 0 & 0 & 1 & -x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} + \frac{P}{EI} \begin{Bmatrix} -\frac{x^3}{6}Y(x) - \frac{(x-l)^3}{6}Y(x-l) - 2x^3 + 5lx^2 \\ -\frac{x^2}{2}Y(x) - \frac{(x-l)^3}{2}Y(x-l) - 2x^2 + 5lx \\ -xY(x) - (x-l)Y(x-l) - 2x \\ Y(x) + Y(x-l) - 2 \end{Bmatrix} \tag{28}$$

At the right end, the free end, for:

$$x = \frac{3l}{2}$$

we can have (29):

$$\begin{Bmatrix} y\left(\frac{3l}{2}\right) \\ \omega\left(\frac{3l}{2}\right) \\ M\left(\frac{3l}{2}\right) \\ T\left(\frac{3l}{2}\right) \end{Bmatrix} = \begin{bmatrix} 1 & \frac{3l}{2} & \frac{1}{2EI}\left(\frac{3l}{2}\right)^2 & -\frac{1}{6EI}\left(\frac{3l}{2}\right)^3 \\ 0 & 1 & \frac{1}{EI}\left(\frac{3l}{2}\right) & -\frac{1}{2EI}\left(\frac{3l}{2}\right)^2 \\ 0 & 0 & 1 & -\frac{3l}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_0 \\ \omega_0 \\ M_0 \\ T_0 \end{Bmatrix} + \{Ve\}_{\frac{3l}{2}} \tag{29}$$

With conditions at the ends of cantilever beam (30):

$$\begin{cases} y_0 = 0 \\ \omega_0 = 0 \\ M\left(\frac{3l}{2}\right) = 0 \\ T\left(\frac{3l}{2}\right) = 0 \end{cases} \tag{30}$$

we have (31):

$$\begin{Bmatrix} y\left(\frac{3l}{2}\right) \\ \omega\left(\frac{3l}{2}\right) \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & \frac{3l}{2} & \frac{1}{2EI}\left(\frac{3l}{2}\right)^2 & -\frac{1}{6EI}\left(\frac{3l}{2}\right)^3 \\ 0 & 1 & \frac{1}{EI}\left(\frac{3l}{2}\right) & -\frac{1}{2EI}\left(\frac{3l}{2}\right)^2 \\ 0 & 0 & 1 & -\frac{3l}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ M_0 \\ T_0 \end{Bmatrix} + \{Ve\}_{\frac{3l}{2}} \tag{31}$$

Now, we can write a linear system of 4 equations with 4 unknowns, its solutions being the unknown elements for the state vector from the origin, respectively, for the state vector of section *2* from the right extremity of the cantilever beam. We have:

- for de state vector at de section *0*, (32):

$$\begin{cases} y_0 = 0 \\ \omega_0 = 0 \\ M_0 = -Pl \\ T_0 = P \end{cases} \tag{32}$$

- for de state vector at de section *2*, (33):

$$\begin{cases} y\left(\frac{3l}{2}\right) = \frac{7Pl^3}{12EI} \\ \omega\left(\frac{3l}{2}\right) = \frac{Pl^2}{2EI} \\ M\left(\frac{3l}{2}\right) = 0 \\ T\left(\frac{3l}{2}\right) = 0 \end{cases} \tag{33}$$

values identical to those obtained after application of classical calculus (Suciu & Tripa, 2021). The maximum cutting force is at aggregation element and body of dental bridge.

The maximum moment is taken by the pole tooth, which was assumed to be an embedded bearing.

Results and Discussion

It was presented in this paper a study about the calculus of cantilever dental bridge with a single pole in medial extension assimilated as a cantilever beam. A medial extension is an extension of anterior part of mouth, in contrast to a distal extension consisting in an extension to posterior part of oral cavity. The cantilever beam model was embedded in the left end and free in the right end, loaded with two concentrated vertical forces, which were assumed to act as follows: one in the middle of the pole, which was considered as an embedded support and the other, in the middle the body of the bridge (Figure 1 and Figure 2). Applications of mathematical algorithm for calculus in dentistry are in continuous development.

In the first part of the work, it was studied an approach for cantilever beam, embedded at the left end and free at the right end with two equal concentrated vertical loads, the model for a cantilever dental bridge with a single pole in medial extension with facilities offered by Dirac's and Heaviside's functions and operators. This idea of studying a dental bridge as a beam with the functions and operators of Dirac and Heaviside is an original idea and gives an elegant and relatively simple solution to the calculus of efforts and deformations and therefore, also of normal and tangential stresses that arise in any section of the cantilever beam, therefore, also of the cantilever dental bridge. With the results obtained for the efforts, the normal and tangential stresses can then be determined in the beam and, by analogy, in the cantilever dental bridge with a single pole in medial extension.

In the second part of the paper, the calculus of the cantilever beam, model for the cantilever dental bridge, through TMM is presented. This approach is a different, but an original idea too. The state vector for a certain section x was defined, the state vector for the left section, the origin section and the state vector for the right end, the last section of the cantilever beam. Applying TMM, a matrix relation was written for a certain section x: the state vector corresponding to section x was written according to the state vector of the origin section, the transfer matrix between the origin and section x, to which a corresponding vector was added for external loads. In this matrix relationship, $x=l$ was made and thus a matrix relationship was obtained between the state vector of the right end section, the last section, and the state vector of the origin section, the left end section. Now, in this last matrix relationship, the conditions can be set in the embedment end, on the left end and on the free end, on the right. After that, a linear system of 4 equations with 4 unknowns was written, its solution allowed the calculus of the unknown elements in the two state vectors corresponding to the two ends of the beam. At this moment, the efforts (bending moment and cutting force) and deformations (arrow and rotation) can be calculated in any section of the cantilever beam and therefore, of the cantilever dental bridge. Knowing the efforts, the normal and tangential stresses can be calculated in any section of the cantilever dental bridge. TMM is based on the facilities offered by calculus with Dirac's and Heaviside's functions and operators, and the great advantage of this method is the fact that the algorithm can be very easily programmed and gives quick results, which can be used even in situ in dentistry and orthodontics, especially in special situations. The maximum cutting force obtained is at aggregation element and body of dental bridge. The maximum moment is taken by the pole tooth, which was assumed to be an embedded bearing, what was expected.

Conclusion

Applications of mathematical algorithm for calculus in dentistry and orthodontics are **i**n continuous development. This paper presents an original and interesting approach for calculus of cantilever dental bridge with a single pole in medial extension. It was presented a calculus of efforts (bending moment and cutting force) and deformations (arrow and rotation) in a cantilever dental bridge with a single pole in medial extension, that is a different, but an original approach for orthodontic problems about cantilever dental bridge studies. We treat this problem

by assimilating the cantilever dental bridge with a cantilever beam, embedded at the left end and free at the right end. It is very important for bio-engineering applications because the calculus of beams is well known by specialists in field. The idea of applying the approach by Transfer-Matrix Method (TMM) is original and very easy and interesting for cantilever dental bridge calculus as a cantilever beam. At first, it was presented a beam calculus using the facilities given by the functions and operators of Dirac and Heaviside. After, we applied the TMM for calculus of cantilever dental bridge with medial extension as a cantilever beam. TMM is very easy to program, and the results can be used rapidly in situ by dentists in special conditions. We hope for the future to present validation of results by Finite Elements Method and by experimental research too.

Conflict of Interest: Does not exist.

References

Burke, F.J.T. & Lucarotti, P.S.K. (2012). Ten year survival of bridges placed in the General Dental Services in England and Wales. *Journal of Dentistry*, *40*(11). https://doi.org/10.1016/j.jdent.2012.07.002.

Dikova, T. & Vasilev, T. (2019). Bending fracture of Co-Cr dental bridges, produced by additive technologies: Simulation analysis and test. *Engineering Fracture Mechanics, 218*. https://doi.org/10.1016/j.engfracmech.2019.106583.

Dikova, T. (2018). Bending fracture of Co-Cr dental bridges, produced by additive technologies: experimental investigation. *Procedia Structural Integrity, 13*, 461-468. https://doi.org/10.1016/j.prostr.2018.12.077.

Gaspar, M. & Weichert, F. (2013). Integrated construction and simulation of tool paths for milling dental crowns and bridges. *Computer-Aided Design, 45*, 1170–1181.

Gery, P.-M. & Calgaro, J.-A. (1987). *Les Matrices-Transfert dans le calcul des structures*. Eyrolles, Paris.

Pjetursson, B. E., Bragger, U., Lang, N. P. & Zwahlen, M. (2007). Comparison of survival and complication rates of tooth-supported fixed dental prostheses (FDPs) and implant-supported FDPs and single crowns. *(SCs). Clin Oral Implants Res.*, *18*(Suppl 3), 97–113. https://doi.org/10.1111/j.1600-0501.2007.01439.x.

Quinna, G. D., Studart, A. R. C., Herbert, C., Ver Hoef, J. R., & Arola, D. (2010). Fatigue of zirconia and dental bridge geometry: Design implications. *Dental Materials, 26*, 11331136. https://doi.org/10.1016/j.dental.2010.07.014.

Roccuzzo, A, Jensen, S. S., Worsaae, N. & Gotfredsen, K. (2020). Implant-supported 2-unit cantilevers compared with single crowns on adjacent implants: a comparative retrospective case series. *J. Prosthet. Dent.*, *123*(5), 717–23. https://doi.org/10.1016/j.prosdent.2019.04.024.

Sharma, A., Rahul, G. R., Poduval, S. T., & Shetty, K. (2012). Assessment of various factors for feasibility of fixed cantilever bridge: A review study. *ISRN Dentistry, 2012*, Article ID 259891. https://doi.org/10.5402/2012/259891.

Styranivska, O., Kliuchkovska, N. & Mykyyevych, N. ((2017). Comparison of using different bridge prosthetic designs for partial defect restoration through mathematical modeling. *Eur J Dent*., *11*(3), 345–351.

Studart, A. R., Fikser, F., Kocher, P, & Gauckler, L. J. (2007). Fatigue of zirconia under cyclic loading in water and its implications for the design of dental bridges. *Dental Materials 23*, 106–114. https://doi.org/10.1016/j.dental.2005.12.008.

Suciu, M. Tripa, M.-S. (2021). *Strength of Materials*. UT Press, Cluj-Napoca, Romania.

Tribst, J. P. M., Dai Piva, A. M. O., Bottino, M. A., Nishioka, R. S., Borges, A. L. S. & Ozcan, M. (2020). Digital image correlation and finite element analysis of bone strain generated by implant-retained cantilever fixed prosthesis. *Eur J Prosthodont Restor Dent*., *28*(1), 10–7. https://doi.org/10.1922/EJPRD_1941Tribst08

Tripa, M., Sorcoi, D., Florescu, C., Sorcoi, A., & Suciu, M. (2020). About the Analytical Calculus of Dental Bridge with Distal Extension and Single Pole Tooth Assimilated as a Beam by Transfer-Matrix Method (TMM). *International Journal of Latest Research in Science and Technology*, *9*(6), 19-22.

Tripa, M., Sorcoi, D., Florescu, I., Sorcoi, L., Paunescu, D., & Suciu, M. (2019). About the study of frames by Transfer-Matrix Method (TMM) – similarity of dental bridges with frames. *Acta Technica Napocensis*, *62*(4), 569-572.

Tripa, M., Sorcoi, D., Florescu, I., Sorcoi, L., Paunescu, D., & Suciu, M. (2019). Contributions on the analytical calculus of simple dental bridge assimilated with a beam embedded at both ends by Transfer-Matrix Method (TMM). *Acta Technica Napocensis*, *62*(4), 573-576.

Warren, C. Y. (1989). *ROARC'S-Formulas for Stress & Strain* (6th ed.). McGraw-Hill Book Company.